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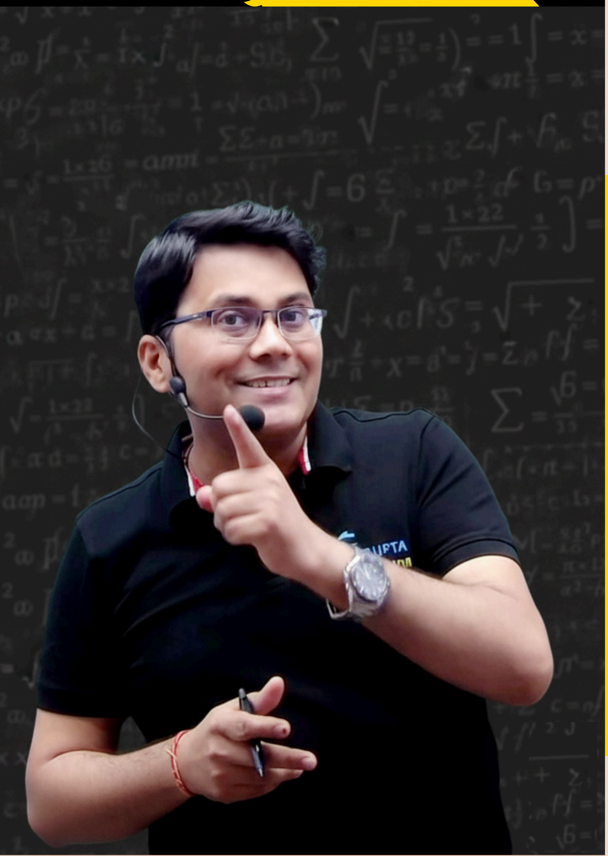

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(BOOKS FOR GRADES XII, XI & X)

MATHMISSION

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2025-2026

📖 Detailed Solutions (PTS-16)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\cot^{-1}(-1)=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{4}\right)\right)+\pi+\left[\pi-\cot^{-1}1\right]$

$\Rightarrow \quad =\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)+\pi+\left(\pi-\frac{\pi}{4}\right)$

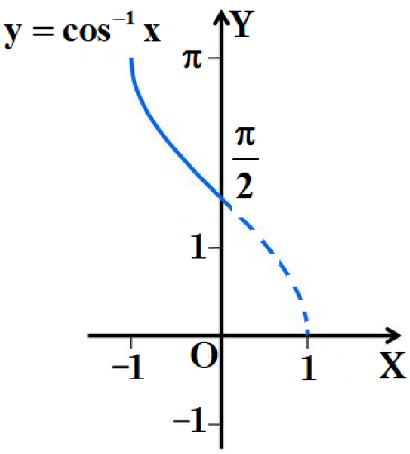
$\Rightarrow \quad =\frac{\pi}{4}+\pi+\frac{3\pi}{4}$

$\Rightarrow \quad =\frac{\pi+3\pi}{4}+\pi$

$\Rightarrow \quad =\frac{4\pi}{4}+\pi=2\pi$.

OR

Graph of the function $f(x)=\cos^{-1}x$, where $x\in[-1,\ 0]$ is given below.



Note that, the dotted portion shown in graph is **not** needed for $f(x)=\cos^{-1}x$ in this question, as we have been given that $x\in[-1,\ 0]$.

Also for the function $f(x)=2\cos^{-1}x,\ x\in[-1,\ 0]$, the range is given by $[\pi,\ 2\pi]$.

Note that, $\frac{\pi}{2}\le\cos^{-1}x\le\pi$, if $-1\le x\le 0$

22. Let one of the number be x so, the other number will be $\frac{9}{x}$.

Let $P=x^2+\frac{81}{x^2}$

$\Rightarrow \frac{dP}{dx}=2x-\frac{162}{x^3}$ and $\frac{d^2P}{dx^2}=2+\frac{486}{x^4}$

For $\frac{dP}{dx}=0,\ 2x-\frac{162}{x^3}=0 \quad \Rightarrow \frac{x^4-81}{x^3}=0 \quad \therefore x=3$

$\because \left(\frac{d^2P}{dx^2}\right)_{\text{at } x=3}=2+\frac{486}{81}=2+6=8>0$

$\therefore$ P is minimum at $x = 3$.

Hence, the numbers are 3, 3.

23. Rewriting the line, $\dfrac{x-\frac{3}{5}}{\frac{1}{5}} = \dfrac{y+\frac{7}{15}}{\frac{1}{15}} = \dfrac{z-\frac{3}{10}}{-\frac{1}{10}}$ i.e., $\dfrac{x-\frac{3}{5}}{6} = \dfrac{y+\frac{7}{15}}{2} = \dfrac{z-\frac{3}{10}}{-3}$

The direction ratios of the line are 6, 2, –3.

Also recall that the direction ratios of parallel lines are proportional.

Therefore, the required equation of line is $\dfrac{x+1}{6} = \dfrac{y-2}{2} = \dfrac{z-3}{-3}$.

24. Consider $x \times y = \sqrt{a^{\tan^{-1} t}} \times \sqrt{a^{\cot^{-1} t}} = \sqrt{a^{\tan^{-1} t + \cot^{-1} t}} = \sqrt{a^{\pi/2}}$

$\Rightarrow x \times y = a^{\pi/4}$

$\Rightarrow x \times \dfrac{dy}{dx} + y \times 1 = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{y}{x}$ ...(i)

Again differentiating w.r.t. x both sides, $\dfrac{d^2y}{dx^2} = -\dfrac{x\dfrac{dy}{dx} - y \times \dfrac{dx}{dx}}{x^2} = -\dfrac{x\left(-\dfrac{y}{x}\right) - y \times 1}{x^2}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{2y}{x^2}$ $\therefore x^2 \dfrac{d^2y}{dx^2} - 2y = 0$.

OR

We have $y = x^{\frac{1}{x}}$

Taking log on both the sides, we get $\log y = \log x^{\frac{1}{x}}$

$\Rightarrow \log y = \dfrac{1}{x} \log x$

$\Rightarrow \dfrac{1}{y} \times \dfrac{dy}{dx} = \dfrac{1}{x} \times \dfrac{1}{x} + \log x \times \left(-\dfrac{1}{x^2}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x^2}[1 - \log x]$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x^{\frac{1}{x}}}{x^2}[1 - \log x]$

At $x = 1$, $\dfrac{dy}{dx} = \dfrac{1^{\frac{1}{1}}}{1^2}[1 - \log 1] = 1[1 - 0] = 1$.

25. $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$

$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a} + \vec{b}|^2$

$\therefore (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$

$\Rightarrow |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$

$\Rightarrow 4\vec{a} \cdot \vec{b} = 0$

$\Rightarrow \vec{a}\cdot\vec{b}=0$

Since $\vec{a}$ and $\vec{b}$ are non-zero vectors so, $\vec{a}\perp\vec{b}$.

Hence, the angle between $\vec{a}$ and $\vec{b}$ is 90° i.e., $\frac{\pi}{2}$.

SECTION C

26. Let $I=\int\frac{\cos x}{\sin 3x}dx=\int\frac{\cos x}{3\sin x-4\sin^3 x}dx$ $\quad$ [Put $\sin x=t\Rightarrow \cos x dx=dt$

$\Rightarrow I=\int\frac{1}{3t-4t^3}dt=\int\frac{1}{t(3-4t^2)}dt=\int\frac{t}{t^2(3-4t^2)}dt$ $\quad$ [Put $t^2=u\Rightarrow t\,dt=\frac{du}{2}$

$\Rightarrow I=\frac{1}{2}\int\frac{1}{u(3-4u)}du$

$\Rightarrow I=\frac{1}{2}\times\frac{1}{3}\int\left[\frac{1}{u}+\frac{4}{(3-4u)}\right]du=\frac{1}{6}\left[\log|u|-\log|3-4u|\right]+c$

$\therefore I=\frac{1}{6}\log\left|\frac{\sin^2 x}{3-4\sin^2 x}\right|+c$.

27. For biased coin, $P(\text{head}):P(\text{tail})=1:3$ i.e., $\frac{P(\text{head})}{P(\text{tail})}=\frac{1}{3}$ i.e., $P(\text{tail})=3\,P(\text{head})$.

As $P(\text{head})+P(\text{tail})=1$ so, $P(\text{head})=\frac{1}{4}$, $P(\text{tail})=\frac{3}{4}$.

Let E : coin shows head; E_1 : the coin is biased; E_2 : the coin is fair.

$\therefore\ P(E_1)=P(E_2)=\frac{1}{2}$, $P(E|E_1)=\frac{1}{4}$, $P(E|E_2)=\frac{1}{2}$.

By Bayes' theorem, $P(E_1|E)=\frac{P(E_1)\,P(E|E_1)}{P(E_1)\,P(E|E_1)+P(E_2)\,P(E|E_2)}$

$\Rightarrow P(E_1|E)=\frac{\frac{1}{4}\times\frac{1}{2}}{\frac{1}{4}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$.

OR

Let E : getting a six.

$\therefore P(E)=\frac{1}{6}$, $P(\bar{E})=1-P(E)=\frac{5}{6}$.

If A starts the game then, he can win in first, third, fifth, … throws.

Therefore, $P(\text{A wins})=P(E)+P(\bar{E}\bar{E}E)+P(\bar{E}\bar{E}\bar{E}\bar{E}E)+...=\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+...$

$\Rightarrow P(\text{A wins})=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{6}{11}$ $\quad$ [$\because a+ar+ar^2+...\infty=\frac{a}{1-r}$, Sum of an infinite G.P.

Also, $P(\text{B wins})=1-P(\text{A wins})=1-\frac{6}{11}=\frac{5}{11}$.

28. We shall use the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

Let $I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx \ldots(i)$

$\Rightarrow I = \int_1^3 \frac{\sqrt{4-(1+3-x)}}{\sqrt{(1+3-x)}+\sqrt{4-(1+3-x)}}dx$

$\Rightarrow I = \int_1^3 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx \ldots(ii)$

On adding (i) and (ii), we get $2I = \int_1^3 \left(\frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}} + \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\right)dx = \int_1^3 1\,dx = [x]_1^3 = 3-1$

$\therefore\ I = 1$.

OR

Let $I = \int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}}dx = \int_1^e \frac{1}{x\sqrt{4-(\log x)^2}}dx$

Put $\log x = t \Rightarrow \frac{dx}{x} = dt$. Also when $x=1 \Rightarrow t=0$ and $x=e \Rightarrow t=1$.

$\therefore I = \int_0^1 \frac{1}{\sqrt{4-t^2}}dt = \left[\sin^{-1}\left(\frac{t}{2}\right)\right]_0^1 = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$\Rightarrow I = \frac{\pi}{6}$.

29. On rewriting the D.E., $\frac{dy}{dx} = \frac{y}{x} - e^{\frac{y}{x}}$.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.

$\therefore\ v + x\frac{dv}{dx} = v - e^v$

$\Rightarrow x\frac{dv}{dx} = -e^v$

$\Rightarrow -\int e^{-v}dv = \int \frac{dx}{x}$

$\Rightarrow e^{-v} = \log|x| + c$

$\Rightarrow e^{-\frac{y}{x}} = \log|x| + c$ or, $\left[\log|x| + c\right] \times e^{\frac{y}{x}} = 1$.

OR

$\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$ $\qquad\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right.$

$\Rightarrow \frac{dy}{dx} = 2\sin x\cos y$

$\Rightarrow \int \sec y\,dy = \int 2\sin x\,dx$

$\Rightarrow \log|\sec y + \tan y| = -2\cos x + C$

It is given that when $x=\frac{\pi}{4},\ y=0$ so, $\log|\sec 0+\tan 0|=-2\cos\frac{\pi}{4}+C$

$\Rightarrow \log|1|=-2\times\frac{1}{\sqrt{2}}+C \qquad \Rightarrow C=\sqrt{2}$

Hence, the particular solution of D.E. is $\log|\sec y+\tan y|=\sqrt{2}-2\cos x$.

30. Let $I=\int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}dx$

$$\Rightarrow I=\int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}\times\frac{\sqrt{2x+1}+\sqrt{2x-1}}{\sqrt{2x+1}+\sqrt{2x-1}}dx$$

$$\Rightarrow I=\int_1^4 \frac{\sqrt{2x+1}+\sqrt{2x-1}}{(2x+1)-(2x-1)}dx=\frac{1}{2}\int_1^4\left[\sqrt{2x+1}+\sqrt{2x-1}\right]dx$$

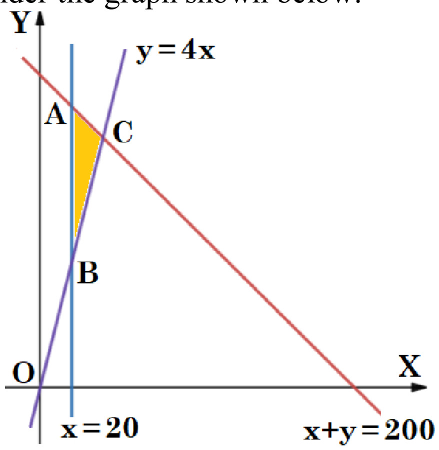
$$\Rightarrow I=\frac{1}{2}\left[\frac{1}{2}\times\frac{2}{3}(2x+1)^{3/2}+\frac{1}{2}\times\frac{2}{3}(2x-1)^{3/2}\right]_1^4$$

$$\Rightarrow I=\frac{1}{2}\times\frac{1}{2}\times\frac{2}{3}\left[(2x+1)^{3/2}+(2x-1)^{3/2}\right]_1^4$$

$$\Rightarrow I=\frac{1}{6}\left\{\left[(9)^{3/2}+(7)^{3/2}\right]-\left[(3)^{3/2}+(1)^{3/2}\right]\right\}=\frac{1}{6}\left\{27+7\sqrt{7}-3\sqrt{3}-1\right\}$$

$$\Rightarrow I=\frac{26+7\sqrt{7}-3\sqrt{3}}{6}.$$

31. Consider the graph shown below.



Corner Points	Value of $z=500x+400y$
A(20, 180)	82000
B(20, 80)	42000 ← Minimum value
C(40, 160)	84000

Therefore, the minimum value of z is 42000 and it is obtained at (20, 80).

SECTION D

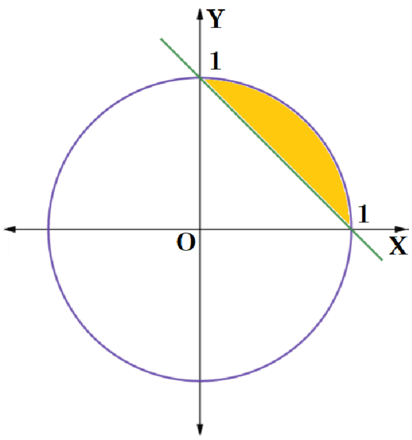
32. Consider $x^2+y^2=1,\ x+y=1$.

On solving, $x^2+(1-x)^2=1$

$\Rightarrow 2x(x-1)=0$

$\Rightarrow x=0, 1$

Points of intersection : $(0,1); (1,0)$.



Required area $=\int_0^1\sqrt{1-x^2}dx-\int_0^1(1-x)dx$

$$=\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_0^1+\frac{1}{2}\left[(1-x)^2\right]_0^1$$

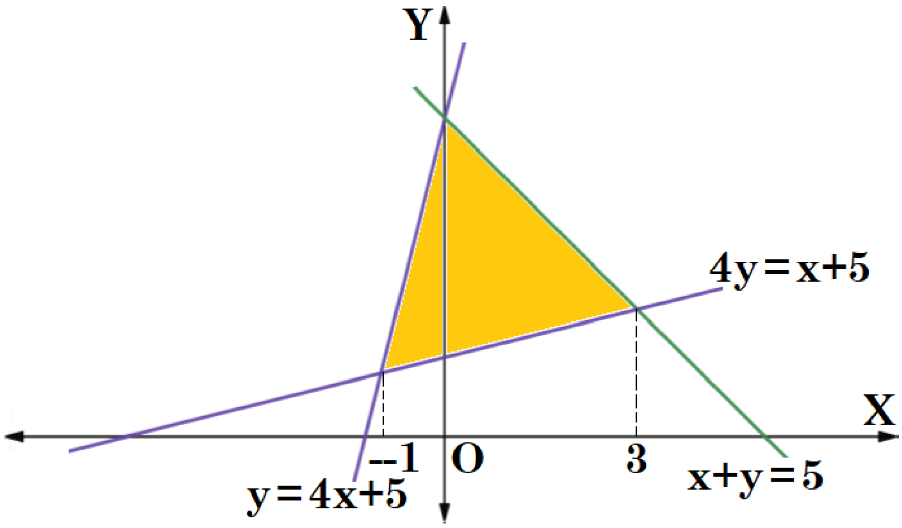
$$=\left[\left(0+\frac{1}{2}\times\frac{\pi}{2}\right)-(0+0)\right]+\frac{1}{2}\left[0^2-1\right]$$

$$=\left(\frac{\pi}{4}-\frac{1}{2}\right)\text{ Sq.units}.$$

OR

The given lines $y=4x+5$, $x+y=5$ and $4y=x+5$ form a triangle.

Required area $=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\frac{1}{4}\int_{-1}^{3}(x+5)dx$



Required area $=\frac{1}{8}\left[(4x+5)^2\right]_{-1}^{0}-\frac{1}{2}\left[(5-x)^2\right]_{0}^{3}-\frac{1}{8}\left[(x+5)^2\right]_{-1}^{3}$

$$=\frac{1}{8}[25-1]-\frac{1}{2}[4-25]-\frac{1}{8}[64-16]$$

$$=3+\frac{21}{2}-6=\frac{15}{2}\text{ Sq.units}.$$

33. Let $x_1, x_2\in[0,\infty)$ and let $f(x_1)=f(x_2)$.

That is, $4x_1^2+4x_1-5=4x_2^2+4x_2-5$

$\Rightarrow(4x_1^2-4x_2^2)+(4x_1-4x_2)=0$

$\Rightarrow(x_1-x_2)(4x_1+4x_2+4)=0$

As $x_1, x_2\in[0,\infty)$ so, $(4x_1+4x_2+4)\neq 0$.

That means, $(x_1-x_2)=0 \Rightarrow x_1=x_2$.

Therefore, $f(x)$ is one-one.

Let $y=f(x),\ y\in[-5,\infty)$

$\Rightarrow y=4x^2+4x-5=(2x)^2+2(2x)(1)+1^2-6$

$\Rightarrow y=(2x+1)^2-6$

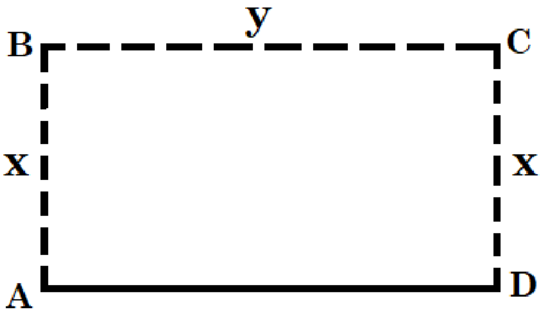
$\Rightarrow y+6=(2x+1)^2$

$\Rightarrow x=\dfrac{\sqrt{y+6}-1}{2}\in[0,\infty)\ \forall\ y\in[-5,\infty)$.

That is, Range = Codomain .

Hence, f is onto also.

34. Consider the diagram shown below.



Here $AB+BC+CD=200$

$\Rightarrow x+y+x=200$

$\Rightarrow 2x+y=200$

Also, area of garden $= xy = x(200-2x)$

Therefore, $A(x)=x(200-2x)$

$\therefore\ A(x)=200x-2x^2$.

Since $A(x)=200x-2x^2$

$\Rightarrow A'(x)=200-4x$

Also, $A''(x)=-4<0$ (Case of maxima)

For $A'(x)=0,\ 200-4x=0 \Rightarrow x=50$

Now maximum value of $A(x)$, $A(50)=200\times 50-2\times 50^2=2\times 50(100-50)=5000\ m^2$.

35. $f(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, $f(-\beta)=\begin{bmatrix}\cos(-\beta) & -\sin(-\beta) & 0\\ \sin(-\beta) & \cos(-\beta) & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\beta & \sin\beta & 0\\ -\sin\beta & \cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}$

Now $f(\alpha)\cdot f(-\beta)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\beta & \sin\beta & 0\\ -\sin\beta & \cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos\alpha\cos\beta+\sin\alpha\sin\beta & \cos\alpha\sin\beta-\sin\alpha\cos\beta & 0\\ \sin\alpha\cos\beta-\cos\alpha\sin\beta & \sin\alpha\sin\beta+\cos\alpha\cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\alpha-\beta) & \sin(\beta-\alpha) & 0\\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0\\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$=f(\alpha-\beta)$.

Hence, $f(\alpha)\cdot f(-\beta)=f(\alpha-\beta)$.

Now, $\text{Det.}[f(\alpha-\beta)]=\begin{vmatrix}\cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0\\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0\\ 0 & 0 & 1\end{vmatrix}$

Expanding along 3rd row, we get $\text{Det.}[f(\alpha-\beta)]=0-0+1[\cos^2(\alpha-\beta)+\sin^2(\alpha-\beta)]$

$\Rightarrow \text{Det.}[f(\alpha-\beta)]=1\times 1$

$\therefore \text{Det.}[f(\alpha-\beta)]=1$.

OR

For $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, $|A|=0+2(0-2)+1(3+2)=1$.

Also $\text{adj.A}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$ $\therefore\ A^{-1}=\dfrac{\text{adj.A}}{|A|}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$

So, $(AB)^{-1}=B^{-1}A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}=\begin{bmatrix}9-1+2 & 6-1+2 & 18-2+5\\ -45+6-10 & -30+6-10 & -90+12-25\\ 15-2+4 & 10-2+4 & 30-4+10\end{bmatrix}$

$$=\begin{bmatrix}10 & 7 & 21\\ -49 & -34 & -103\\ 17 & 12 & 36\end{bmatrix}.$$

SECTION E

36. **(i)** Here $P(E_1)=50\%=0.5,\ P(E_2)=20\%=0.2,\ P(E_3)=30\%=0.3$.

Also $P(A|E_1)=0.06,\ P(A|E_2)=0.04,\ P(A|E_3)=0.03$.

As $P(E_2\cap A)=P(E_2)\times P(A|E_2)$

So, $P(E_2\cap A)=(0.2)(0.04)=0.008$.

(ii) As $P(E_1\cap A)=P(E_1)\times P(A|E_1)$

$\Rightarrow P(E_1\cap A)=(0.5)(0.06)=0.03$.

(iii) Required probability $=1-P(E_1|A)$

$$\Rightarrow\quad =1-\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$\Rightarrow\quad =1-\frac{(0.5)(0.06)}{(0.5)(0.06)+(0.2)(0.04)+(0.3)(0.03)}$$

$$\Rightarrow\quad =1-\frac{0.030}{0.047}$$

$$\Rightarrow\quad =\frac{17}{47}.$$

OR

(iii) As $P(E_3\cap A)=P(E_3)\times P(A|E_3)$

$\Rightarrow P(E_3\cap A)=(0.3)(0.03)=0.009$.

Also, $P(A)=P(E_1)\ P(A|E_1)+P(E_2)\ P(A|E_2)+P(E_3)\ P(A|E_3)$

So, $P(A)=(0.5)(0.06)+(0.2)(0.04)+(0.3)(0.03)$

$\Rightarrow P(A)=0.030+0.008+0.009$

$\Rightarrow P(A)=0.047$.

And, $\sum_{i=1}^{3} P(E_i|A)=P(E_1|A)+P(E_2|A)+P(E_3|A)$

$$\Rightarrow\quad =\left(\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}+\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}+\frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}\right)$$

$$\Rightarrow \quad = \frac{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$\Rightarrow \quad =1$.

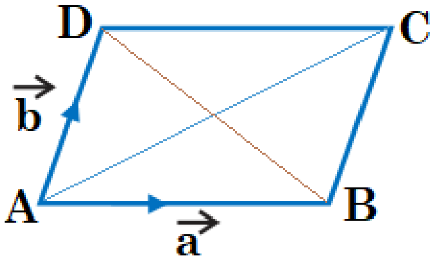
37. **(i)** Since the vectors representing the sides of triangle are taken in order, therefore $\vec{p}+\vec{q}+\vec{r}=\vec{0}$.

So, $(\vec{q}+\vec{r})=-\vec{p}$.

Also it is given that, $(\vec{q}+\vec{r})=k(\vec{p})$.

On comparing, we get $k=-1$.

(ii) Consider the figure shown.



Since in a parallelogram opposite sides are equal in length and are parallel. Then $\overrightarrow{AD}=\overrightarrow{BC}=\vec{b}$.

In ΔABC, $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

$\Rightarrow \overrightarrow{AC}=\vec{a}+\vec{b}=2\hat{i}-4\hat{j}-5\hat{k}+2\hat{i}+2\hat{j}+3\hat{k}$

$\therefore \overrightarrow{AC}=4\hat{i}-2\hat{j}-2\hat{k}$.

(iii) Refer to **(ii)**. In ΔABC, $\overrightarrow{AC}=4\hat{i}-2\hat{j}-2\hat{k}$.

Also in ΔABD, $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}$

$\Rightarrow \vec{b}=\vec{a}+\overrightarrow{BD}$

$\therefore \overrightarrow{BD}=\vec{b}-\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}-2\hat{i}+4\hat{j}+5\hat{k}=6\hat{j}+8\hat{k}$

Therefore, $\overrightarrow{AC}+\overrightarrow{BD}=4\hat{i}-2\hat{j}-2\hat{k}+6\hat{j}+8\hat{k}=4\hat{i}+4\hat{j}+6\hat{k}$.

OR

(iii) For a ΔABC, $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

$\Rightarrow \overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=(-5\hat{i}+2\hat{j}+3\hat{k})-(-2\hat{i}-3\hat{j}+5\hat{k})$

$\Rightarrow \overrightarrow{BC}=-3\hat{i}+5\hat{j}-2\hat{k}$

Therefore, $\left|\overrightarrow{BC}\right|=\sqrt{(-3)^2+5^2+(-2)^2}=\sqrt{38}$.

38. **(i)** Since $h(t)$ is a polynomial function, so it is continuous everywhere when $t \geq 0$.

(ii) $h(t)=-\frac{7}{2}t^2+\frac{13}{2}t+1$ implies, $h'(t)=-7t+\frac{13}{2}$

and, $h''(t)=-7<0$ (case of maxima)

For $h'(t)=0,\ -7t+\frac{13}{2}=0 \Rightarrow t=\frac{13}{14}$ seconds.

🕮 Detailed Solutions (PTS-17)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. Consider LHS and put $\theta = \frac{1}{2}\cos^{-1}\frac{a}{b} \quad \Rightarrow \cos 2\theta = \frac{a}{b}$ ...(i)

LHS : Let $y = \tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$

$\Rightarrow y = \tan\left[\frac{\pi}{4} + \theta\right] + \tan\left[\frac{\pi}{4} - \theta\right]$

$\Rightarrow y = \dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} + \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}$

$\Rightarrow y = \dfrac{1 + \tan\theta}{1 - \tan\theta} + \dfrac{1 - \tan\theta}{1 + \tan\theta}$

$\Rightarrow y = \dfrac{(1 + \tan\theta)^2 + (1 - \tan\theta)^2}{1 - \tan^2\theta}$

$\Rightarrow y = \dfrac{2(1 + \tan^2\theta)}{1 - \tan^2\theta} = 2\left(\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)$

$\Rightarrow y = 2\left(\dfrac{1}{\cos 2\theta}\right) = 2\left(\dfrac{1}{a/b}\right)$ [By (i)]

$\therefore y = \dfrac{2b}{a} = \text{RHS}.$

22. Let the required point be P (x, y).

According to question, $\dfrac{dy}{dt} = 8\left(\dfrac{dx}{dt}\right)$ …(i)

Now $6y = x^3 + 2$

$\Rightarrow 6\left(\dfrac{dy}{dt}\right) = 3x^2\dfrac{dx}{dt}$

On using (i), we get : $6\left(8\dfrac{dx}{dt}\right) = 3x^2\left(\dfrac{dx}{dt}\right)$

$\Rightarrow 3x^2 = 48$

$\Rightarrow x^2 = 16$

$\therefore x = \pm 4.$

When $x=4 \Rightarrow 6y=4^3+2 \Rightarrow y=11$

When $x=-4 \Rightarrow y=-\frac{31}{3}$.

So, the required points are (4, 11) and $\left(-4,-\frac{31}{3}\right)$.

OR

Given $f(x)=\log\cos x$

$\Rightarrow f'(x)=-\frac{\sin x}{\cos x}=-\tan x$

$\because \tan x>0$ in $x\in\left(0,\frac{\pi}{2}\right)$ $\therefore -\tan x<0$ in $x\in\left(0,\frac{\pi}{2}\right)$ $\Rightarrow f'(x)<0\ \forall x\in\left(0,\frac{\pi}{2}\right)$

So, the function f (x) is strictly decreasing on $x\in\left(0,\frac{\pi}{2}\right)$.

Also, $\tan x<0$ in $x\in\left(\frac{\pi}{2},\pi\right)$ $\therefore -\tan x>0$ in $x\in\left(\frac{\pi}{2},\pi\right)$ $\Rightarrow f'(x)>0\ \forall x\in\left(\frac{\pi}{2},\pi\right)$

So, the function f (x) is strictly increasing on $x\in\left(\frac{\pi}{2},\pi\right)$.

23. Unit vector along $\lambda\hat{i}+6\hat{j}-2\hat{k}$ is $\frac{\lambda\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+40}}$.

Note that $(\hat{i}+\hat{j}+\hat{k})\times(\lambda\hat{i}+6\hat{j}-2\hat{k})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ \lambda & 6 & -2\end{vmatrix}=-8\hat{i}+(2+\lambda)\hat{j}+(6-\lambda)\hat{k}$ ...(i)

So, $\left|(\hat{i}+\hat{j}+\hat{k})\times\left(\frac{\lambda\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+40}}\right)\right|=\sqrt{2}$

$\Rightarrow \left|(\hat{i}+\hat{j}+\hat{k})\times(\lambda\hat{i}+6\hat{j}-2\hat{k})\right|=\sqrt{2}\times\sqrt{\lambda^2+40}$

By (i), we get : $\sqrt{64+(2+\lambda)^2+(6-\lambda)^2}=\sqrt{2}\times\sqrt{\lambda^2+40}$

$\Rightarrow 64+4+\lambda^2+4\lambda+36+\lambda^2-12\lambda=2\lambda^2+80$

$\Rightarrow -8\lambda=-24$

$\Rightarrow \lambda=\frac{24}{8}$

$\therefore \lambda=3$.

OR

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=\vec{0}.\vec{0}$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow 1^2+1^2+1^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\therefore \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{3}{2}$.

24. $y=\cos(m\cos^{-1}x)$

$$\Rightarrow \frac{dy}{dx}=-\sin(m\cos^{-1}x)\times m\times\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx}=m\sin(m\cos^{-1}x)$$

On differentiating again w.r.t. x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2}-\frac{2x}{2\sqrt{1-x^2}}\times\frac{dy}{dx}=m\cos(m\cos^{-1}x)\times\frac{-m}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-m^2y$$

$$\therefore\ (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0.$$

25. Rewriting the lines, $\frac{x-1}{-3}=\frac{y-7}{p}=\frac{z-3}{2};\ \frac{x-1}{-3p}=\frac{y-5}{1}=\frac{z-6}{-5}.$

Since these lines are perpendicular so, $(-3)(-3p)+p\times 1+2(-5)=0$

$\Rightarrow 9p+p-10=0$

$\Rightarrow 10p=10$

$\Rightarrow p=\frac{10}{10}$

$\therefore p=1.$

SECTION C

26. Let $I=\int\frac{\cos x-\sin x}{\sqrt{10+\sin 2x}}dx$

$$\Rightarrow I=\int\frac{\cos x-\sin x}{\sqrt{9+(1+\sin 2x)}}dx$$

$$\Rightarrow I=\int\frac{\cos x-\sin x}{\sqrt{9+(\sin x+\cos x)^2}}dx$$

Put $\sin x+\cos x=t\Rightarrow(\cos x-\sin x)dx=dt$

$$\therefore I=\int\frac{dt}{\sqrt{9+t^2}}$$

$$\Rightarrow I=\log\left|t+\sqrt{9+t^2}\right|+C$$

$$\Rightarrow I=\log\left|(\sin x+\cos x)+\sqrt{9+(\sin x+\cos x)^2}\right|+C$$

$$\Rightarrow I=\log\left|(\sin x+\cos x)+\sqrt{10+\sin 2x}\right|+C.$$

OR

Let $I=\int\frac{x+\sin x}{1+\cos x}dx=\int\frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\left(\frac{x}{2}\right)}dx$

$$\Rightarrow I=\frac{1}{2}\int x\sec^2\left(\frac{x}{2}\right)dx+\int\tan\left(\frac{x}{2}\right)dx$$

$$\Rightarrow I=\frac{1}{2}\left[x\int \sec^2\left(\frac{x}{2}\right)dx-\int\left(\frac{d}{dx}(x)\int \sec^2\left(\frac{x}{2}\right)dx\right)dx\right]+\int \tan\left(\frac{x}{2}\right)dx$$

$$\Rightarrow I=\frac{1}{2}\left[x\tan\left(\frac{x}{2}\right).2-\int 2\tan\left(\frac{x}{2}\right)dx\right]+\int \tan\left(\frac{x}{2}\right)dx$$

$$\Rightarrow I=x\tan\left(\frac{x}{2}\right)+C.$$

27. Let E_1 : selection of a person of blood group 'O',

E_2 : selection of a person of other blood group,

E : selection of a person left handed person.

Given that $P(E_1)=30\%,\ P(E\,|\,E_1)=6\%,\ P(E_2)=70\%,\ P(E\,|\,E_2)=10\%$.

So, $P(E)=P(E_1)\ P(E\,|\,E_1)+P(E_2)\ P(E\,|\,E_2)$

$$\Rightarrow P(E)=\frac{30}{100}\times\frac{6}{100}+\frac{70}{100}\times\frac{10}{100}$$

$$\therefore\ P(E)=\frac{11}{125}.$$

OR

Let A and B be the event that the student appears for NEET and JEE Main respectively.
We have $P(A)=30\%,\ P(B)=25\%$ and $P(A\cap B)=15\%$.

(i) $P(A\,|\,B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{15\%}{25\%}=\dfrac{15}{25}=\dfrac{3}{5}$

(ii) $P(B\,|\,A)=\dfrac{P(B\cap A)}{P(A)}=\dfrac{15\%}{30\%}=\dfrac{15}{30}=\dfrac{1}{2}$.

28. Consider $I=\int e^{2x}.\sin\left(\frac{\pi}{4}+x\right)dx$

$$\Rightarrow I=\sin\left(\frac{\pi}{4}+x\right)\int e^{2x}dx-\int\left(\frac{d}{dx}\sin\left(\frac{\pi}{4}+x\right)\int e^{2x}dx\right)dx$$

$$\Rightarrow I=\sin\left(\frac{\pi}{4}+x\right)\times\frac{e^{2x}}{2}-\int\cos\left(\frac{\pi}{4}+x\right)\times\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\sin\left(\frac{\pi}{4}+x\right)\times\frac{e^{2x}}{2}-\cos\left(\frac{\pi}{4}+x\right)\int\frac{e^{2x}}{2}dx+\int\left(\frac{d}{dx}\cos\left(\frac{\pi}{4}+x\right)\int\frac{e^{2x}}{2}dx\right)dx$$

$$\Rightarrow I=\frac{1}{2}e^{2x}\sin\left(\frac{\pi}{4}+x\right)-\frac{1}{4}e^{2x}\cos\left(\frac{\pi}{4}+x\right)-\frac{1}{4}\int e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$$

$$\Rightarrow I=\frac{1}{2}e^{2x}\sin\left(\frac{\pi}{4}+x\right)-\frac{1}{4}e^{2x}\cos\left(\frac{\pi}{4}+x\right)-\frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I=\frac{1}{4}e^{2x}\left[2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]$$

$$\Rightarrow I=\frac{1}{5}e^{2x}\left[2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]$$

Now $\displaystyle\int_0^{\pi}e^{2x}.\sin\left(\frac{\pi}{4}+x\right)dx=\frac{1}{5}\left[e^{2x}\left\{2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right\}\right]_0^{\pi}$

$\Rightarrow \quad =\frac{1}{5}\left[e^{2\pi}\left\{2\sin\left(\frac{\pi}{4}+\pi\right)-\cos\left(\frac{\pi}{4}+\pi\right)\right\}\right]-\frac{1}{5}\left[e^{0}\left\{2\sin\left(\frac{\pi}{4}+0\right)-\cos\left(\frac{\pi}{4}+0\right)\right\}\right]$

$\Rightarrow \quad =\frac{1}{5}\left[e^{2\pi}\left\{2\times\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right\}\right]-\frac{1}{5}\left[\left\{2\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right\}\right]$

$\Rightarrow \quad =-\frac{1}{5\sqrt{2}}(e^{2\pi}+1)$.

29. We have $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$\Rightarrow \sqrt{(1+x^2)(1+y^2)}+xy\frac{dy}{dx}=0$

$\Rightarrow \int\frac{\sqrt{1+x^2}}{x}dx+\int\frac{y}{\sqrt{1+y^2}}dy=0$

$\Rightarrow I_1+\frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}dy=0...(i)$

Consider $I_1=\int\frac{\sqrt{1+x^2}}{x}dx$

Put $x^2+1=t^2 \Rightarrow dx=\frac{t\,dt}{\sqrt{t^2-1}}$

$\therefore I_1=\int\frac{t}{\sqrt{t^2-1}}\frac{t}{\sqrt{t^2-1}}dt$

$\Rightarrow I_1=\int\frac{t^2}{t^2-1}dt$

$\Rightarrow I_1=\int\left(1+\frac{1}{t^2-1}\right)dt$

$\Rightarrow I_1=t+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|$

$\Rightarrow I_1=\sqrt{x^2+1}+\frac{1}{2}\log\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right|$

Substituting the value of I_1 in (i), we get : $\sqrt{x^2+1}+\frac{1}{2}\log\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right|+\frac{1}{2}\left[2\sqrt{1+y^2}\right]=C$

$\therefore \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|+\sqrt{1+x^2}+\sqrt{1+y^2}=C$ is the required solution.

OR

$(3xy+y^2)dx+(x^2-xy)dy=0$

$\Rightarrow (3xy+y^2)dx=(xy-x^2)dy$

$\Rightarrow \frac{dy}{dx}=\frac{3xy+y^2}{xy-x^2}$

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = \frac{3x^2v + v^2x^2}{x^2v - x^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{3v + v^2}{v-1} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{4v}{v-1}$$

$$\Rightarrow \int \frac{v-1}{v}dv = \int \frac{4dx}{x}$$

$$\Rightarrow v - \log|v| = 4\log|x| + C$$

$$\Rightarrow \frac{y}{x} - \log\left|\frac{y}{x}\right| = 4\log|x| + C$$

$$\Rightarrow \frac{y}{x} - \log|y| + \log|x| = 4\log|x| + C$$

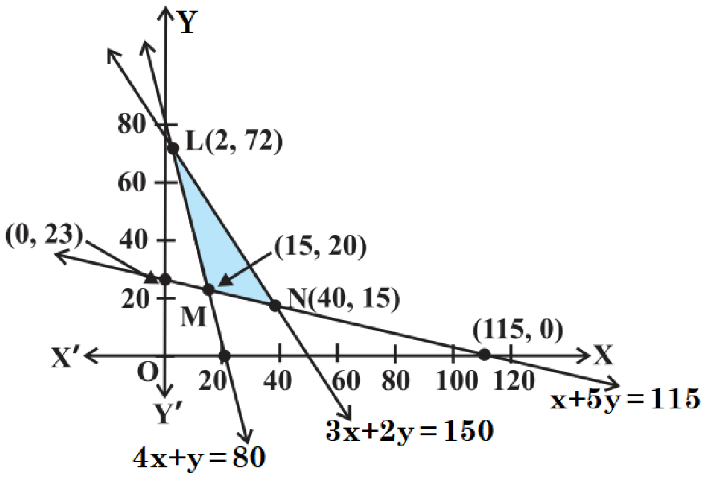
$$\therefore \frac{y}{x} = 3\log|x| + \log|y| + C.$$

30. Graph is shown on the next page.

Corner Points	Value of Z
L(2, 72)	76
M(15, 20)	50 ← Minimum
N(40, 15)	95

Therefore, the minimum value of Z is 50.

Also the maximum value of Z occurs at the point (40, 15).



31. $$\int \frac{\cos(x+a)}{\sin(x+b)}dx = \int \frac{\cos\{(x+b)+(a-b)\}}{\sin(x+b)}dx$$

$$\Rightarrow \quad = \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)}dx$$

$$\Rightarrow \quad = \int \left\{\frac{\cos(x+b)\cos(a-b)}{\sin(x+b)} - \frac{\sin(x+b)\sin(a-b)}{\sin(x+b)}\right\}dx$$

$$\Rightarrow \quad = \int \{\cos(a-b)\cot(x+b) - \sin(a-b)\}\,dx$$

$\Rightarrow \quad = \cos(a-b)\log|\sin(x+b)| - x\sin(a-b) + C$.

SECTION D

32. We have $(x-2)^2 + y^2 = 4$ ...(i)

Centre is at (2, 0) and radius is 2 units.

$y = x-2$ ...(ii)

Solving (i) and (ii), we get

$(x-2)^2 + (x-2)^2 = 4$

$\Rightarrow 2(x-2)^2 = 4$

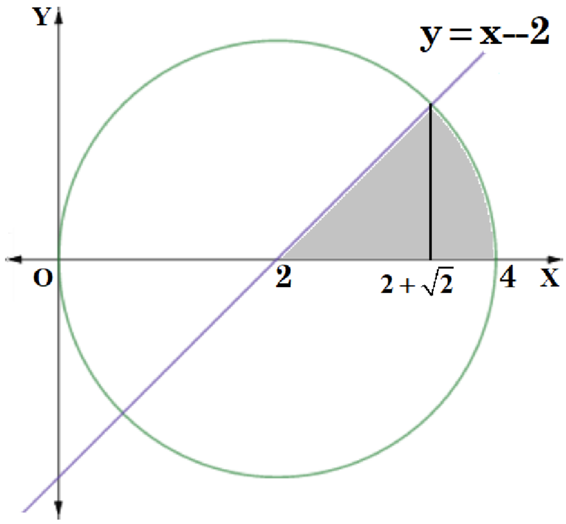
$\Rightarrow (x-2)^2 = 2$

$\Rightarrow x = 2 \pm \sqrt{2}$

So, $y = 2 \pm \sqrt{2} - 2$

$\Rightarrow y = \pm\sqrt{2}$

$\therefore$ Points of intersection : $(2\pm\sqrt{2}, \pm\sqrt{2})$.



Required area $= \int_2^{2+\sqrt{2}} (x-2)dx + \int_{2+\sqrt{2}}^{4} \sqrt{4-(x-2)^2}\,dx$

$\Rightarrow \quad = \frac{1}{2}\left[(x-2)^2\right]_2^{2+\sqrt{2}} + \left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + \frac{4}{2}\sin^{-1}\frac{x-2}{2}\right]_{2+\sqrt{2}}^{4}$

$\Rightarrow \quad = \frac{1}{2}\left[(2+\sqrt{2}-2)^2 - (2-2)^2\right] + \left[0 + 2\times\frac{\pi}{2}\right] - \left[\frac{\sqrt{2}}{2}\times\sqrt{2} + 2\times\frac{\pi}{4}\right]$

$\Rightarrow \quad = 1 + \pi - \left[1 + \frac{\pi}{2}\right]$

$\Rightarrow \quad = \frac{\pi}{2}$ Sq.units .

OR

Consider the integral $\int_a^b f(x)\,dx$.

Let $x = a+b-t \Rightarrow dx = -dt$.

Also when $x = a \Rightarrow t = b$ and, when $x = b \Rightarrow t = a$.

So, $\int_a^b f(x)\,dx = \int_b^a f(a+b-t)(-dt)$

$\Rightarrow \int_a^b f(x)\,dx = -\left[-\int_a^b f(a+b-t)dt\right]$

$\Rightarrow \int_a^b f(x)\,dx = \int_a^b f(a+b-t)dt$

Hence, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$. [Replacing t by x

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$...(i)

$$\Rightarrow I=\int_0^{\pi/2}\frac{\frac{\pi}{2}-x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\cos x+\sin x}dx-\int_0^{\pi/2}\frac{x}{\cos x+\sin x}dx$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\sin x\frac{1}{\sqrt{2}}\right)}dx-I \qquad \text{[By (i)}$$

$$\Rightarrow 2I=\frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\left(\log\left|\sec\left(\frac{\pi}{2}-\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right)\right|\right)-\left(\log\left|\sec\left(0-\frac{\pi}{4}\right)+\tan\left(0-\frac{\pi}{4}\right)\right|\right)\right]$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right]$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right|\right]$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[2\log(\sqrt{2}+1)\right]$$

$$\therefore I=\frac{\pi\log(\sqrt{2}+1)}{2\sqrt{2}}.$$

33. We have $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\frac{x}{x^2+1}$, $\forall\ x\in\mathbb{R}$.

Let $m, n\in\mathbb{R}$ so that $f(m)=f(n)$.

That implies, $\frac{m}{m^2+1}=\frac{n}{n^2+1}$

$\Rightarrow mn^2+m=nm^2+n$
$\Rightarrow m-n=mn(m-n)$
$\Rightarrow m-n=mn(m-n)$
$\Rightarrow (mn-1)(m-n)=0$

Either $m-n=0$ or, $mn-1=0$ i.e., $m=n$ or, $m=\frac{1}{n}$.

So, $f(m)=f(n)$ does not necessarily imply $m=n$ for all $m, n\in\mathbb{R}$.
Hence $f(x)$ is not one-one.

Let $y = f(x) = \dfrac{x}{x^2+1}$

$\Rightarrow yx^2 - x + y = 0$

$\Rightarrow x = \dfrac{1 \pm \sqrt{1-4y^2}}{2y}$

Now for x to be real, we must have $1-4y^2 \ge 0$ i.e., $(1-2y)(1+2y) \ge 0$ i.e., $y \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

That is for all $x \in \mathbb{R}$ (domain), we do not have $y \in \mathbb{R}$ (codomain) i.e., Range $\neq$ Codomain. Hence $f(x)$ is not onto.

34. The equation of line through $(3,2,1)$ and $(4,\lambda,5)$ is $L_1 : \dfrac{x-3}{1} = \dfrac{y-2}{\lambda-2} = \dfrac{z-1}{4} = m$ (say).

Also the line through $(4,2,-2)$ and $(6,5,-1)$ is $L_2 : \dfrac{x-4}{2} = \dfrac{y-2}{3} = \dfrac{z+2}{1} = s$ (say).

$\therefore$ Coordinates of any random points on the lines L_1 and L_2 are $P(m+3,\ m\lambda-2m+2,\ 4m+1)$ and $Q(2s+4,\ 3s+2,\ s-2)$ respectively.

Since the lines L_1 and L_2 intersect each other so, the points P and Q must coincide with each other for some values of m and s.

That is, $m+3 = 2s+4 ...(i), m\lambda - 2m + 2 = 3s + 2 ...(ii), 4m+1 = s-2 ...(iii)$

Solving (i) and (iii) we get : $s = -1, m = -1$.

Replacing values of m and s in (ii), we have : $-\lambda + 2 + 2 = -3 + 2$

$\therefore \lambda = 5$.

OR

As it is given that $\vec{a} = \vec{b} + \vec{c}$ $\quad \therefore p\hat{i} + q\hat{j} + r\hat{k} = s\hat{i} + 3\hat{j} + 4\hat{k} + 3\hat{i} + \hat{j} - 2\hat{k}$

$\Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

On equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ on both sides, we get : $p = s+3,\ q = 4,\ r = 2$.

Also $ar(ABC) = 5\sqrt{6} = \dfrac{1}{2}|\vec{a} \times \vec{b}|$

$\Rightarrow 10\sqrt{6} = \text{Magnitude of} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & q & r \\ s & 3 & 4 \end{vmatrix}$

$\Rightarrow 10\sqrt{6} = \left|\hat{i}(4q-3r) - \hat{j}(4p-rs) + \hat{k}(3p-sq)\right|$

$\Rightarrow 10\sqrt{6} = \left|\hat{i}(16-6) - \hat{j}(4p-2s) + \hat{k}(3p-4s)\right|$

$\Rightarrow 10\sqrt{6} = \sqrt{10^2 + (12+2s)^2 + (9-s)^2}$

$\Rightarrow 600 = 100 + 144 + 48s + 4s^2 + 81 - 18s + s^2$

$\Rightarrow 5s^2 + 30s - 275 = 0$

$\Rightarrow s^2 + 6s - 55 = 0$

$\Rightarrow (s+11)(s-5) = 0$

Either $(s+11) = 0$ or, $(s-5) = 0$

$\Rightarrow s = -11,\ 5$

$\therefore p = -8,\ 8$.

Hence, $p = -8, q = 4, r = 2, s = -11$ or, $p = 8, q = 4, r = 2, s = 5$.

35. Let the number of children be x and the amount distributed by Seema for one child be ₹y.

So, $(x-8)(y+10)=xy$

$\Rightarrow 5x-4y=40$...(i)

and $(x+16)(y-10)=xy$

$\Rightarrow 5x-8y=-80$...(ii)

To solve (i) and (ii), let $A=\begin{pmatrix}5 & -4\\ 5 & -8\end{pmatrix}$, $B=\begin{pmatrix}40\\ -80\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\end{pmatrix}$

$\because AX=B \Rightarrow X=A^{-1}B$

Now $A^{-1}=\dfrac{1}{-40+20}\begin{pmatrix}-8 & 4\\ -5 & 5\end{pmatrix}=\dfrac{1}{20}\begin{pmatrix}8 & -4\\ 5 & -5\end{pmatrix}$

$\therefore X=\dfrac{1}{20}\begin{pmatrix}8 & -4\\ 5 & -5\end{pmatrix}\begin{pmatrix}40\\ -80\end{pmatrix}$

$\Rightarrow \begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}32\\ 30\end{pmatrix}$

Clearly, $x=32,\ y=30$.

Hence the number of children $=32$ and, the amount distributed by Seema $=$₹30.

SECTION E

36. **(i)** Consider the diagram.

We have the perimeter of the field, $x+2y+x+\dfrac{1}{2}(2\pi y)=1000$

$\Rightarrow 2x+2y+\pi y=1000$.

Also, $y=\dfrac{1000-2x}{2+\pi}$.

(ii) Note that the playground is rectangular so, area (A) is given by (x) (2y).

That is, $A=2x\left(\dfrac{1000-2x}{2+\pi}\right)$ [Refer to (i)

$\Rightarrow \quad =\dfrac{2}{2+\pi}(1000x-2x^2)$.

(iii) We have $A=\dfrac{2}{2+\pi}(1000x-2x^2)$

$\Rightarrow \dfrac{dA}{dx}=\dfrac{2}{2+\pi}(1000-4x)$ and, $\dfrac{d^2A}{dx^2}=-\dfrac{8}{2+\pi}<0$ so, A is maximum.

For points of local maxima/minima, we have $\dfrac{dA}{dx}=0,\ \dfrac{2}{2+\pi}(1000-4x)=0$

$\Rightarrow x=250$ m.

Also, the maximum area of the playground alone, $A=\dfrac{2}{2+\pi}(1000\times 250-2\times 250^2)$

$\Rightarrow A=\dfrac{250000}{2+\pi}$ m^2.

OR

(iii) Total area including the parking space is, $S=x(2y)+\dfrac{1}{2}\pi y^2$

$\Rightarrow S=\dfrac{2}{2+\pi}(1000x-2x^2)+\dfrac{1}{2}\pi\left(\dfrac{1000-2x}{2+\pi}\right)^2$

$\Rightarrow \dfrac{dS}{dx}=\dfrac{2}{2+\pi}(1000-4x)+\pi\left(\dfrac{1000-2x}{2+\pi}\right)\left(-\dfrac{2}{2+\pi}\right)$

and, $\dfrac{d^2S}{dx^2}=-\dfrac{8}{2+\pi}+\dfrac{4\pi}{(2+\pi)^2}$

$\Rightarrow \dfrac{d^2S}{dx^2}=-\dfrac{16+4\pi}{(2+\pi)^2}<0$ so, S is maximum.

For points of local maxima/minima, we have $\dfrac{dS}{dx}=0$

$\Rightarrow \dfrac{2}{2+\pi}(1000-4x)+\pi\left(\dfrac{1000-2x}{2+\pi}\right)\left(-\dfrac{2}{2+\pi}\right)=0$

$\Rightarrow (1000-4x)=\pi\left(\dfrac{1000-2x}{2+\pi}\right)$

$\Rightarrow \dfrac{2\pi x}{2+\pi}-4x=\dfrac{1000\pi}{2+\pi}-1000$

$\Rightarrow x=\dfrac{1000}{\pi+4}$ m.

37. **(i)** Let P(x, y) be the position of the fighter-jet and the soldier is placed at A(3, 7).

$\therefore AP=\sqrt{(x-3)^2+(y-7)^2}$ …(i)

Also $y=x^2+7$

$\Rightarrow x^2=y-7$ …(ii)

Using (i) and (ii), we get : $AP=\sqrt{(x-3)^2+x^4}$

$\therefore f(x)=\sqrt{(x-3)^2+x^4}$

(ii) As $u=[f(x)]^2$ $\therefore u=\left[\sqrt{(x-3)^2+x^4}\right]^2$

$\Rightarrow u=(x-3)^2+x^4$

$\Rightarrow \dfrac{du}{dx}=2(x-3)+4x^3=4x^3+2x-6$.

(iii) $\because \dfrac{du}{dx}=2(x-3)+4x^3$ $\therefore \dfrac{d^2u}{dx^2}=2+12x^2$.

For $\dfrac{du}{dx}=0$, $2(x-3)+4x^3=0$ $\Rightarrow x=1$

$\because \dfrac{d^2u}{dx^2}(\text{at } x=1)=14>0$

$\therefore$ u is minimum when $x=1$.

Also the least distance is, $AP=\sqrt{(1-3)^2+1^4}=\sqrt{5}$ units.

OR

(iii) Since soldier shoots when the fighter-jet is nearest to him. So, we need to minimize f (x).

Now $f(x)=\sqrt{(x-3)^2+x^4}$

As $u=\left[f(x)\right]^2$ $\therefore u=\left[\sqrt{(x-3)^2+x^4}\right]^2$ $\Rightarrow u=(x-3)^2+x^4$

{When u is minimum, then f (x) is also minimum}.

Now $\frac{du}{dx}=2(x-3)+4x^3$ and $\frac{d^2u}{dx^2}=2+12x^2$.

For $\frac{du}{dx}=0$, $2(x-3)+4x^3=0$ $\Rightarrow x=1$

$\because \frac{d^2u}{dx^2}(\text{at } x=1)=14>0$

$\therefore$ u is minimum when $x=1$. That means, f (x) is also minimum.

So $x=1$, when replaced in the parabolic curve $y=x^2+7$ gives $y=1^2+7=8$.

Hence, the fighter-jet will be at (1, 8) when the soldier shoots it down.

38. **(i)** Let E : Shruti is selected and F : Hina is selected.

$\therefore P(E)=\frac{1}{3}$, $P(F)=\frac{1}{2}$.

So, P(at most one of them is selected) $=P(E)P(F')+P(E')P(F)+P(E')P(F')$

$\Rightarrow$ $=\frac{1}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{2}=\frac{5}{6}$.

(ii) P(both of them are selected) $=P(E)P(F)=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$.

Also, P(at least one of them is selected) $=P(E)P(F')+P(E')P(F)+P(E)P(F)$

$\Rightarrow$ $=\frac{1}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{2}=\frac{2}{3}$.

Detailed Solutions (PTS-18)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. Put $x=\sin\theta \Rightarrow \theta=\sin^{-1}x$. As $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$ so, $\theta\in\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$ and $3\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$\therefore \sin^{-1}(3x-4x^3)=\sin^{-1}(3\sin\theta-4\sin^3\theta)$

$=\sin^{-1}\sin 3\theta=3\theta$

$=3\sin^{-1}x$.

OR

Since the school is boys school, that clearly means no student of the school can be sister of any student of the school. Hence, $R=\phi$, showing that R is the empty relation.

It is also obvious that the difference between heights of any two students of the school has to be less than 3 meters.

This shows that $R'=A\times A$ is the universal relation.

22. Since surface area of the sphere is, $S=4\pi r^2$

$\Rightarrow \frac{dS}{dt}=4\pi\times 2r\times\frac{dr}{dt}$

As $\frac{dS}{dt}=\frac{dr}{dt}$ so, $\frac{dr}{dt}=4\pi\times 2r\times\frac{dr}{dt}$

$\Rightarrow 4\pi\times 2r=1 \quad \Rightarrow r=\frac{1}{8\pi}$

So, volume of the sphere is given by $V=\frac{4\pi}{3}\times r^3$

$\Rightarrow V=\frac{4\pi}{3}\times\left(\frac{1}{8\pi}\right)^3$

$\Rightarrow V=\frac{4\pi}{3}\times\frac{1}{8\pi}\times\frac{1}{8\pi}\times\frac{1}{8\pi}$

$\therefore V=\frac{1}{384\pi^2}$ cubic units.

23. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=(2\hat{i}-2\hat{j}+3\hat{k})\times(-\hat{k})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -2 & 3\\ 0 & 0 & -1\end{vmatrix}$

$\Rightarrow (\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=2\hat{i}+2\hat{j}$.

OR

The direction ratios of the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z+17}{6}$ are 2, 3, 6.

The vector parallel to the given line is $2\hat{i}+3\hat{j}+6\hat{k}$.

Let θ be the required angle between line and vector $10\hat{i}+2\hat{j}-11\hat{k}$.

So, $\cos\theta=\frac{(10\hat{i}+2\hat{j}-11\hat{k}).(2\hat{i}+3\hat{j}+6\hat{k})}{\sqrt{10^2+2^2+(-11)^2}\sqrt{2^2+3^2+6^2}}$

$\Rightarrow \cos\theta=\frac{20+6-66}{15\times7}=-\frac{40}{15\times7}=-\frac{8}{21}$

$\therefore \theta=\cos^{-1}\left(-\frac{8}{21}\right)$ or, $\pi-\cos\left(\frac{8}{21}\right)$.

24. $x^y+y^x=a^b$ can be written as $e^{\log x^y}+e^{\log y^x}=a^b$

$\Rightarrow e^{y\log x}+e^{x\log y}=a^b$

$\Rightarrow e^{y\log x}\times\left(y\times\frac{1}{x}+\log x\times\frac{dy}{dx}\right)+e^{x\log y}\times\left(x\times\frac{1}{y}\times\frac{dy}{dx}+\log y\times1\right)=0$

$\Rightarrow x^y\times\left(\frac{y}{x}+\log x\times\frac{dy}{dx}\right)+y^x\times\left(\frac{x}{y}\times\frac{dy}{dx}+\log y\right)=0$

$\Rightarrow yx^{y-1}+x^y\times\log x\times\frac{dy}{dx}+y^{x-1}\times x\times\frac{dy}{dx}+y^x\times\log y=0$

$\Rightarrow\left\{x^y\times\log x+y^{x-1}\times x\right\}\times\frac{dy}{dx}=-\left\{yx^{y-1}+y^x\times\log y\right\}$

$\therefore \frac{dy}{dx}=-\left\{\frac{yx^{y-1}+y^x\times\log y}{x^y\times\log x+y^{x-1}\times x}\right\}$.

25. $\overrightarrow{AB}=3\hat{i}+(m-3)\hat{j}-2\hat{k}$, $\overrightarrow{AC}=9\hat{i}-3\hat{j}-6\hat{k}$

Since the points A, B and C are collinear so, we must have $\frac{3}{9}=\frac{m-3}{-3}=\frac{-2}{-6}$.

Considering $\frac{3}{9}=\frac{m-3}{-3}$

$\Rightarrow m-3=-1$

$\therefore m=2$.

SECTION C

26. Let $I=\int\frac{x+3}{(x+1)(x+4)}dx$

$\Rightarrow I=\int\frac{(x+1)+2}{(x+1)(x+4)}dx$

$\Rightarrow I=\int\frac{(x+1)}{(x+1)(x+4)}dx+\int\frac{2}{(x+1)(x+4)}dx$

$\Rightarrow I=\int\frac{1}{(x+4)}dx+\frac{2}{3}\int\left[\frac{1}{x+1}-\frac{1}{x+4}\right]dx$

$\Rightarrow I=\log|x+4|+\frac{2}{3}\left[\log|x+1|-\log|x+4|\right]+C$

$\Rightarrow I = \log|x+4| + \frac{2}{3}\log\left|\frac{x+1}{x+4}\right| + C$ or, $\frac{1}{3}\log|x+4| + \frac{2}{3}\log|x+1| + C$.

27. Let $P(B) = p$.

As A and B are independent events, so $P(A \cap B) = P(A)P(B)$.

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B)\,[1 - P(A)]$

$\Rightarrow P(A \cup B) = P(A) + P(B)\,P(\bar{A})$

$\Rightarrow 0.6 = 0.4 + p\,[1 - 0.4]$

$\Rightarrow 0.2 = p \times 0.6$

$\therefore p = \frac{1}{3}$.

Now $P(B) = \frac{1}{3}$, $P(B') = 1 - P(B) = \frac{2}{3}$.

OR

Let A be the event that the construction job will be completed on time, and B be the event that there will be a strike.

We have $P(B) = 0.65$, $P(B') = 0.35$, $P(A|B) = 0.32$, $P(A|B') = 0.80$

So, $P(A) = P(B)\,P(A|B) + P(B')\,P(A|B')$

$\Rightarrow \quad = 0.65 \times 0.32 + 0.35 \times 0.80$

$\Rightarrow \quad = 0.488$.

28. Let $I = \int_0^{\pi/2} \left(\frac{5\sin x + 3\cos x}{\sin x + \cos x}\right) dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \left(\frac{5\sin\left(\frac{\pi}{2} - x\right) + 3\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\right) dx$$

$\Rightarrow I = \int_0^{\pi/2} \left(\frac{5\cos x + 3\sin x}{\cos x + \sin x}\right) dx$ ...(ii)

On adding (i) and (ii), we get : $2I = \int_0^{\pi/2} \left(\frac{5\cos x + 3\sin x}{\cos x + \sin x} + \frac{5\sin x + 3\cos x}{\sin x + \cos x}\right) dx$

$\Rightarrow 2I = \int_0^{\pi/2} \left(\frac{8\cos x + 8\sin x}{\cos x + \sin x}\right) dx$

$\Rightarrow I = 4\int_0^{\pi/2} dx = 4[x]_0^{\pi/2}$

$\Rightarrow I = 4\left[\frac{\pi}{2} - 0\right]$

$\therefore I = 2\pi$.

OR

Let $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$\Rightarrow I=\int_0^{\pi/2}\frac{\sec^2 x\ dx}{a^2+b^2\tan^2 x}$

Put $\tan x=t\Rightarrow \sec^2 x\ dx=dt$.

Also when $x=0\Rightarrow t=0$ and when $x\to\frac{\pi}{2}\Rightarrow t\to\infty$.

$\therefore I=\int_0^{\infty}\frac{dt}{a^2+b^2t^2}=\int_0^{\infty}\frac{dt}{a^2+(b\,t)^2}$

$\Rightarrow I=\frac{1}{b}\times\frac{1}{a}\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$

$\Rightarrow I=\frac{1}{ab}\left[\frac{\pi}{2}-0\right]$

$\therefore I=\frac{\pi}{2ab}$.

29. We have $\frac{dy}{dx}=-4xy^2$

$\Rightarrow \int -\frac{dy}{y^2}=\int 4x\,dx$

$\Rightarrow \frac{1}{y}=4\times\frac{x^2}{2}+C$

$\Rightarrow \frac{1}{y}=2x^2+C$

As it is given that $y=1$, when $x=0$ so, $\frac{1}{1}=2\times 0^2+C\ \Rightarrow C=1$.

Therefore, the solution is $\frac{1}{y}=2x^2+1$ i.e., $y=\frac{1}{2x^2+1}$.

OR

Re-write the D.E., $\frac{dy}{dx}=\frac{x+y}{x-y}$ …(i)

Let $f(x,y)=\frac{x+y}{x-y}$

Now $f(\lambda x,\lambda y)=\frac{\lambda x+\lambda y}{\lambda x-\lambda y}=\lambda^0\left(\frac{x+y}{x-y}\right)=\lambda^0\ f(x,y)$.

Therefore, $f(x,y)$ is a homogeneous function of degree zero.

So, the given differential equation is a homogeneous differential equation.

To solve it, substitute $y=vx\ \Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$.

$\therefore v+x\frac{dv}{dx}=\frac{1+v}{1-v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1+v}{1-v}-v$ i.e., $x\frac{dv}{dx}=\frac{1+v^2}{1-v}$

$\Rightarrow \int\frac{v-1}{v^2+1}dv=-\int\frac{dx}{x}$

$$\Rightarrow \frac{1}{2}\int \frac{2v}{v^2+1}dv - \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x}$$

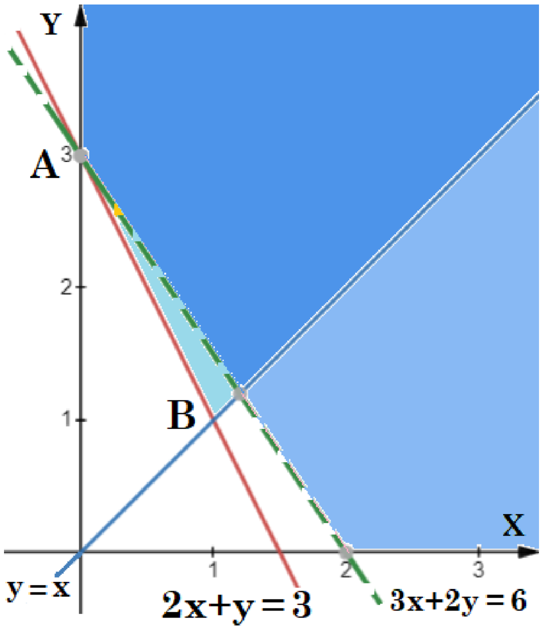
$$\Rightarrow \frac{1}{2}\log\left|v^2+1\right| - \tan^{-1} v = -\log|x| + C$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{y^2}{x^2}+1\right| - \tan^{-1}\frac{y}{x} = -\log|x| + C$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{y^2+x^2}{x^2}\right| - \tan^{-1}\frac{y}{x} = -\log|x| + C$$

$$\Rightarrow \frac{1}{2}\log\left|y^2+x^2\right| - \tan^{-1}\frac{y}{x} = C.$$

30. Consider the graph shown below.



Corner points	Value of Z
A(0, 3)	6 ← Max.
B(1, 1)	5

Note that, $Z = 6$ may or may not be the maximum value, (as the feasible region is unbounded).

To check, let $3x + 2y > 6$.

Note that $3x + 2y > 6$ has **common points with the feasible region** (shown with dark shades).

So, $Z = 3x + 2y$ does not have any maximum value subject to the given constraints.

Therefore, $Z = 6$ is not the maximum value.

31. Let $f(x) = \frac{1}{\log x}$ $\quad \Rightarrow f'(x) = -\frac{1}{(\log x)^2} \times \frac{1}{x} = -\frac{1}{x(\log x)^2}$

By using $\int e^x\left[f(x) + f'(x)\right]dx = e^x\ f(x) + C$, we get

$$\int e^x\left[\frac{1}{\log x} - \frac{1}{x(\log x)^2}\right]dx = \int e^x\left[\frac{1}{\log x} + \left(-\frac{1}{x(\log x)^2}\right)\right]dx = \frac{1}{\log x}\times e^x + C \text{ or, } \frac{e^x}{\log x} + C.$$

SECTION D

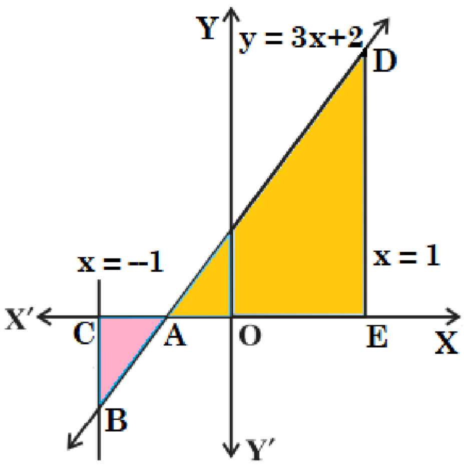
32. Consider the graph shown here.

Required area $= ar(ACBA) + ar(ADEOA)$

$$= \left|\int_{-1}^{-2/3}(3x+2)dx\right| + \int_{-2/3}^{1}(3x+2)dx$$

$$= \left|\left[\frac{(3x+2)^2}{2\times 3}\right]_{-1}^{-2/3}\right| + \left[\frac{(3x+2)^2}{2\times 3}\right]_{-2/3}^{1}$$

$$= \left|0 - \frac{1}{6}\right| + \left[\frac{25}{6} - 0\right] = \frac{13}{3}\text{ Sq. units}.$$

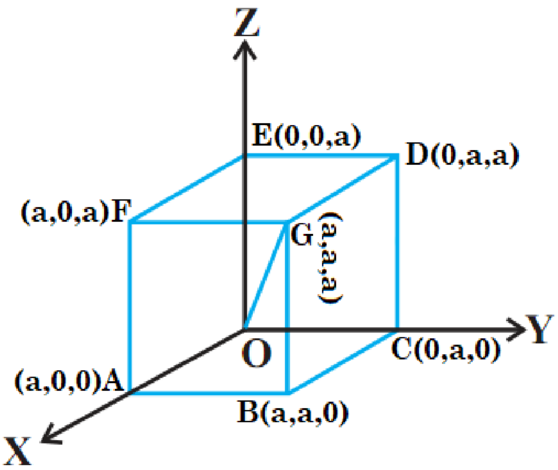


33. Every line L_1 is parallel to itself. So, $(L_1, L_1) \in R \ \forall \ L_1 \in L$.
That means, R is reflexive.
Let $(L_1, L_2) \in R \ \forall \ L_1, L_2 \in L$.
That means, L_1 is parallel to L_2. It further implies that, L_2 is parallel to L_1.
Then, $(L_2, L_1) \in R$. So, R is symmetric.
Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R \ \forall \ L_1, L_2, L_3 \in L$.
That means, L_1 is parallel to L_2 and, L_2 is parallel to L_3.
Clearly it implies that, L_1 is parallel to L_3.
Then, $(L_1, L_3) \in R$. So, R is transitive.
As R is reflexive, symmetric and transitive so, R is equivalence relation.

OR

Since $x \in \left[0, \frac{\pi}{2}\right]$ i.e., x lies in the first quadrant in which $f(x) = \sin x$ will always have unique value for unique x.
That means, f is one-one.
Similarly in $x \in \left[0, \frac{\pi}{2}\right]$, $g(x) = \cos x$ will always have unique value for unique x.
That means, g is one-one.
Now $f(x) + g(x) = \sin x + \cos x = h(x)$ say
As $h\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{2}$ and $h\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} + \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{2}$.
Clearly, $h\left(\frac{\pi}{6}\right) = h\left(\frac{\pi}{3}\right)$ but $\frac{\pi}{6} \neq \frac{\pi}{3}$.
So, $h(x)$ is not one-one.
Recall that, $[-1, 1]$ is the range of $f(x) = \sin x$ and $g(x) = \cos x$ both.
That is, $[-1, 1] \neq \mathbb{R}$ which implies $\text{range} \neq \text{codomain}$.
So, $f(x)$ and $g(x)$ both are not onto.

34. Consider the figure shown below.



As shown in the figure, the diagonals of the cube of side length a units, are OG, AD, BE and CF with their respective direction ratios as a, a, a; $-a, a, a$; $-a, -a, a$; $a, -a, a$.

Let the given line has A, B, C as its direction ratios.

So, $\cos\alpha = \frac{Aa + Ba + Ca}{\sqrt{A^2 + B^2 + C^2}\sqrt{a^2 + a^2 + a^2}} = \frac{1}{\sqrt{3}} \frac{A + B + C}{\sqrt{A^2 + B^2 + C^2}}$,

$\cos\beta = \frac{1}{\sqrt{3}}\frac{-A+B+C}{\sqrt{A^2+B^2+C^2}}$, $\cos\gamma = \frac{1}{\sqrt{3}}\frac{-A-B+C}{\sqrt{A^2+B^2+C^2}}$ and $\cos\omega = \frac{1}{\sqrt{3}}\frac{A-B+C}{\sqrt{A^2+B^2+C^2}}$.

Now squaring both the sides of these four equations and then adding, we get :

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\omega = \frac{1}{3}\times\frac{(A+B+C)^2}{A^2+B^2+C^2}+\frac{1}{3}\times\frac{(-A+B+C)^2}{A^2+B^2+C^2}+ \frac{1}{3}\times\frac{(-A-B+C)^2}{A^2+B^2+C^2}+\frac{1}{3}\times\frac{(A-B+C)^2}{A^2+B^2+C^2}$$

$$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\omega = \frac{1}{3(A^2+B^2+C^2)}\begin{bmatrix} A^2+B^2+C^2+2AB+2BC+2CA \\ +A^2+B^2+C^2-2AB+2BC-2CA \\ +A^2+B^2+C^2+2AB-2BC-2CA \\ +A^2+B^2+C^2-2AB-2BC+2CA \end{bmatrix}$$

$$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\omega = \frac{1}{3(A^2+B^2+C^2)}\left[4(A^2+B^2+C^2)\right]$$

$$\therefore \cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\omega = \frac{4}{3}.$$

OR

Given parametric equations of line are $x = 3\lambda,\ y = -6\lambda,\ z = 6\lambda$.

So, Cartesian equation is $\frac{x-1}{3} = \frac{y}{-6} = \frac{z}{6}$ i.e., $\frac{x-1}{1} = \frac{y}{-2} = \frac{z}{2}$.

Also, the vector equation is $\vec{r} = \hat{i}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$.

The position vector of a point P on the line, when $\lambda = -1$ is $\vec{r} = \hat{i}+(-1)(\hat{i}-2\hat{j}+2\hat{k}) = 2\hat{j}-2\hat{k}$.

That is, $\overrightarrow{OP} = 2\hat{j}-2\hat{k}$.

Note that, P(0, 2, –2).

The required distance covered by insect, $OP = \sqrt{(0-0)^2+(2-0)^2+(-2-0)^2} = 2\sqrt{2}$ units.

35. The given system of equations $x+y+z=6,\ y+3z=11,\ x-2y+z=0$ can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}.$$

Now $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 1(1+6)-0+1(3-1) = 9 \neq 0 \ \therefore A^{-1}$ exists.

Consider A_{ij} as the cofactor of a_{ij}.

$A_{11}=7,\ A_{12}=3,\ A_{13}=-1;$
$A_{21}=-3,\ A_{22}=0,\ A_{23}=3;$
$A_{31}=2,\ A_{32}=-3,\ A_{33}=1$

$$\therefore \text{adj.}A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Since $AX = B$ implies that, $X = A^{-1}B$

$$\Rightarrow X = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By equality of matrices, we get : $x = 1,\ y = 2,\ z = 3$.

SECTION E

36. **(i)** As given volume is, $1000 = \frac{1}{2}\pi r^2 h$

$$\Rightarrow h = \frac{2000}{\pi r^2}.$$

(ii) The total surface area is given as, $A = \pi r h + \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 + 2rh$

$$\Rightarrow A = \frac{2000}{r} + \pi r^2 + \frac{4000}{\pi r}.$$ [Using $h = \frac{2000}{\pi r^2}$

(iii) $A = \frac{2000}{r} + \pi r^2 + \frac{4000}{\pi r}$

$$\therefore \frac{dA}{dr} = -\frac{2000}{r^2} + 2\pi r - \frac{4000}{\pi r^2}.$$

For $\frac{dA}{dr} = 0$, $2\left(\pi r - \frac{1000}{r^2} - \frac{2000}{\pi r^2}\right) = 0$

$$\Rightarrow \pi^2 r^3 = 1000\pi + 2000$$

$$\therefore r = \left[\frac{1000(\pi + 2)}{\pi^2}\right]^{1/3}.$$

Also, $\frac{d^2A}{dr^2} = \frac{4000}{r^3} + 2\pi + \frac{8000}{\pi r^3}$.

OR

(iii) $A = \frac{2000}{r} + \pi r^2 + \frac{4000}{\pi r}$

$$\therefore \frac{dA}{dr} = -\frac{2000}{r^2} + 2\pi r - \frac{4000}{\pi r^2} \text{ and, } \frac{d^2A}{dr^2} = \frac{4000}{r^3} + 2\pi + \frac{8000}{\pi r^3}.$$

For $\frac{dA}{dr} = 0$, $2\left(\pi r - \frac{1000}{r^2} - \frac{2000}{\pi r^2}\right) = 0$

$$\Rightarrow \pi^2 r^3 = 1000\pi + 2000$$

$$\therefore r = \left[\frac{1000(\pi + 2)}{\pi^2}\right]^{1/3}.$$

$$\because \left.\frac{d^2A}{dr^2}\right]_{\text{at } r = \left[\frac{1000(\pi+2)}{\pi^2}\right]^{1/3}} = \frac{4\pi^2}{(\pi + 2)} + 2\pi + \frac{8\pi}{(\pi + 2)} > 0$$

$$\therefore A \text{ is minimum at } r = 10\left[\frac{(\pi + 2)}{\pi^2}\right]^{1/3}.$$

$\because h = \dfrac{2000}{\pi r^2}$

$\Rightarrow \dfrac{h}{2r} = \dfrac{2000}{2\pi r^3}$

$\Rightarrow \dfrac{h}{2r} = \dfrac{1000\pi^2}{\pi \times 1000(\pi + 2)}$

$\Rightarrow \dfrac{h}{2r} = \dfrac{\pi}{(\pi + 2)}$

Hence, $(2r) : h = (\pi + 2) : \pi$.

37. **(i)** $P(x) = -5x^2 + 125x + 37500$

$\Rightarrow P'(x) = -10x + 125,\ P''(x) = -10$

For $P'(x) = 0,\ -10x + 125 = 0 \ \Rightarrow x = 12.5$

As $P''(12.5) = -10 < 0$, so $x = 12.5$ is the production of company for maximum profit.

(ii) Maximum profit, $P(12.5) = -5(12.5)^2 + 125(12.5) + 37500$

$\Rightarrow P(12.5) =$ ₹38281.25.

(iii) We have $P'(x) = -10x + 125$ and for $P'(x) = 0,\ x = 12.5$

Since $P'(x) > 0$ for $x \in (0, 12.5)$ so, P(x) is strictly increasing in $x \in (0, 12.5)$.

Also, $P'(x) < 0$ for $x \in (12.5, 20)$ so, P(x) is strictly decreasing in $x \in (12.5, 20)$.

OR

(iii) Since $P(x) = -5x^2 + 125x + 37500$, $P(2) = -5(2)^2 + 125(2) + 37500 = -20 + 250 + 37500$

$\therefore P(2) =$ ₹37730.

Therefore, profit is ₹37730 when 2 units are produced in the company.

Also, $P(x) = -5x^2 + 125x + 37500$

$\Rightarrow 38250 = -5x^2 + 125x + 37500$

$\Rightarrow 5x^2 - 125x + 750 = 0$

$\Rightarrow x^2 - 25x + 150 = 0$

$\Rightarrow (x - 15)(x - 10) = 0$

$\therefore x = 10, 15$

As $x > 10$ so, $x = 15$.

Therefore, if the profit is ₹38250, then the production of company is 15 units.

38. Let E_1, E_2 and E_3 be the events that the person A, B and C respectively carries out the task.

Let E be the event that the time taken is more than the allotted time.

$\because P(E_3) = 45\%,\ P(E_3) = 35\%,\ P(E_3) = 20\%,\ P(E|E_1) = \dfrac{1}{16},\ P(E|E_2) = \dfrac{1}{10},\ P(E|E_3) = \dfrac{1}{20}$.

(i) Using total probability theorem, we get

$P(E) = P(E_1) \times P(E|E_1) + P(E_2) \times P(E|E_2) + P(E_3) \times P(E|E_3)$

$\Rightarrow P(E) = \dfrac{45}{100} \times \dfrac{1}{16} + \dfrac{35}{100} \times \dfrac{1}{10} + \dfrac{20}{100} \times \dfrac{1}{20}$

$\Rightarrow P(E) = \dfrac{45}{1600} + \dfrac{35}{1000} + \dfrac{20}{2000}$

$\Rightarrow P(E) = \dfrac{45}{1600} + \dfrac{90}{2000}$

$\Rightarrow P(E) = \frac{5}{100} \times \left(\frac{9}{16} + \frac{9}{10} \right)$

$\Rightarrow P(E) = \frac{9}{20} \times \frac{26}{160}$

$\Rightarrow P(E) = \frac{9}{20} \times \frac{13}{80}$

$\therefore P(E) = \frac{117}{1600}$.

(ii) Using Bayes' theorem, $P(E_2 | E) = \frac{P(E_2) \times P(E | E_2)}{P(E)}$

$\Rightarrow P(E_2 | E) = \frac{\frac{35}{100} \times \frac{1}{10}}{\frac{117}{1600}}$

$\Rightarrow P(E_2 | E) = \frac{\frac{35}{10}}{\frac{117}{16}}$

$\Rightarrow P(E_2 | E) = \frac{35}{10} \times \frac{16}{117} = \frac{7}{2} \times \frac{16}{117}$

$\therefore P(E_2 | E) = \frac{56}{117}$.

Detailed Solutions (PTS-19)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. $\cot^{-1} x = \cos^{-1}(-1) - \text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \cot^{-1} x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\Rightarrow x = \cot\frac{2\pi}{3} = \cot\left(\pi - \frac{\pi}{3}\right) = -\cot\frac{\pi}{3} = -\frac{1}{\sqrt{3}}$.

Now $\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$.

OR

(i) The function $f(x)$ is defined when $5x-3 \le -1$ or $5x-3 \ge 1$

$\Rightarrow 5x \le 2$ or $5x \ge 4$

$\therefore x \le \frac{2}{5}$ or $x \ge \frac{4}{5}$

Hence, the domain of $f(x)$ is $x \in \left(-\infty, \frac{2}{5}\right] \cup \left[\frac{4}{5}, \infty\right)$.

(ii) Since $0 \le \cos^{-1}\left(\frac{1}{2x-1}\right) \le \pi$ i.e., $3\times 0 \le 3\cos^{-1}\left(\frac{1}{2x-1}\right) \le 3\pi$

$\Rightarrow 0-2 \le 3\cos^{-1}\left(\frac{1}{2x-1}\right) - 2 \le 3\pi - 2$

$\Rightarrow -2 \le f(x) \le 3\pi - 2$

Hence, the range of $f(x)$ is $[-2, 3\pi-2]$.

22. Since C is a skew-symmetric matrix so, $C = \frac{1}{2}(A - A')$.

For $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$, $A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

So, $C = \frac{1}{2}\left(\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right)$

$\therefore C=\begin{bmatrix}0 & 2 & -2\\ -2 & 0 & -2\\ 2 & 2 & 0\end{bmatrix}$.

23. Differentiate y with respect to x, $\dfrac{dy}{dx}=4(e^{\sec x}+x)^3\times\dfrac{d}{dx}(e^{\sec x}+x)$

$\therefore \dfrac{dy}{dx}=4(e^{\sec x}+x)^3\times(e^{\sec x}\sec x\tan x+1)$.

24. Substituting $\vec{v}=k(\hat{q}+\hat{r})$ in $\hat{p}.\vec{v}=\hat{q}.\vec{v}$ to get $\hat{p}.\left[k(\hat{q}+\hat{r})\right]=\hat{q}.\left[k(\hat{q}+\hat{r})\right]$

$\Rightarrow \hat{p}.\hat{q}+\hat{p}.\hat{r}=\hat{q}.\hat{q}+\hat{q}.\hat{r}$

$\Rightarrow \hat{p}.\hat{q}+\hat{p}.\hat{r}=1+\hat{q}.\hat{r}$

$\Rightarrow 1-\hat{p}.\hat{q}+\hat{q}.\hat{r}-\hat{p}.\hat{r}=0$

$\Rightarrow \hat{p}.\hat{p}-\hat{p}.\hat{q}+\hat{q}.\hat{r}-\hat{p}.\hat{r}=0$

$\Rightarrow \hat{p}.(\hat{p}-\hat{q})-\hat{r}.(\hat{p}-\hat{q})=0$

$\Rightarrow (\hat{p}-\hat{q}).(\hat{p}-\hat{r})=0$ or, $(\hat{p}-\hat{r}).(\hat{p}-\hat{q})=0$.

OR

Area of QRST $=\left|\overrightarrow{RQ}\times\overrightarrow{RS}\right|=\left|\vec{a}\times\vec{b}\right|$ ...(i)

Also, the area of QRTP $=\left|\overrightarrow{RQ}\times\overrightarrow{RT}\right|$

$=\left|\overrightarrow{RQ}\times(\overrightarrow{RQ}+\overrightarrow{QT})\right|$

$=\left|\vec{a}\times(\vec{a}+\vec{b})\right|$

$=\left|(\vec{a}\times\vec{a})+(\vec{a}\times\vec{b})\right|$

$=\left|\vec{0}+\vec{a}\times\vec{b}\right|=\left|(\vec{a}\times\vec{b})\right|$ ...(ii)

Hence by using (i) and (ii), we can conclude that the area of the parallelogram QRST is equal to the area of parallelogram QRTP.

25. If the lines intersect each other, then the bomb may hit the plane.
Also, if the lines intersect then, S.D. $=0$.

Now $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}_1=2\hat{i}+3\hat{j}+4\hat{k},\ \vec{a}_2=4\hat{i}+\hat{j},\ \vec{b}_2=5\hat{i}+2\hat{j}+\hat{k}$

$\Rightarrow \vec{a}_2-\vec{a}_1=3\hat{i}-\hat{j}-3\hat{k},\ \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 5 & 2 & 1\end{vmatrix}=-5\hat{i}+18\hat{j}-11\hat{k}$

As $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(3\hat{i}-\hat{j}-3\hat{k}).(-5\hat{i}+18\hat{j}-11\hat{k})=-15-18+33=0$.

So, S.D. $=0$ therefore, the bomb may hit the fighter-jet.

SECTION C

26. We have $u=e^{\sin^{-1}\theta}$ and $v=e^{-\cos^{-1}\theta}$.

$\therefore \dfrac{du}{d\theta}=e^{\sin^{-1}\theta}\times\dfrac{1}{\sqrt{1-\theta^2}}$ and $\dfrac{dv}{d\theta}=e^{-\cos^{-1}\theta}\times\dfrac{1}{\sqrt{1-\theta^2}}$

Now $\dfrac{du}{dv}=\dfrac{du/d\theta}{dv/d\theta}=\dfrac{e^{\sin^{-1}\theta}}{e^{-\cos^{-1}\theta}}=e^{\sin^{-1}\theta+\cos^{-1}\theta}=e^{\frac{\pi}{2}}$, which is a constant. Hence, the statement is true.

OR

Given that $\dfrac{x^m}{y^n}=(xy)^{(m-n)}$.

Taking logarithm on both sides, we get $m(\log x)-n(\log y)=(m-n)(\log x+\log y)$

Differentiating both sides w.r.t. x, we get $\dfrac{m}{x}-\dfrac{n}{y}\times\dfrac{dy}{dx}=(m-n)\left(\dfrac{1}{x}+\dfrac{1}{y}\times\dfrac{dy}{dx}\right)$

$$\Rightarrow \frac{m}{x}-\frac{m-n}{x}=\frac{dy}{dx}\times\left[\frac{m-n}{y}+\frac{n}{y}\right]$$

$$\Rightarrow \frac{m}{x}-\frac{m}{x}+\frac{n}{x}=\frac{dy}{dx}\times\left[\frac{m}{y}-\frac{n}{y}+\frac{n}{y}\right]$$

$$\therefore \frac{dy}{dx}=\frac{ny}{mx}.$$

27. Since $(3x-1)f(x)=\dfrac{d}{dx}\left(3x^4-\dfrac{13}{3}x^3+\dfrac{3}{2}x^2+C\right)$

$\Rightarrow (3x-1)f(x)=12x^3-13x^2+3x$

$\Rightarrow (3x-1)f(x)=x(3x-1)(4x-3)$

Hence, $f(x)=(4x^2-3x)$.

Put $x=6$ in $f(x)$, $f(6)=6(4\times 6-3)=126$.

28. Let $I=\displaystyle\int_{\pi/4}^{\pi/2}\operatorname{cosec}^6 x\ \cot^2 x\,dx=\int_{\pi/4}^{\pi/2}\cot^2 x\,(1+\cot^2 x)^2\operatorname{cosec}^2 x\,dx$

Put $\cot x=u \Rightarrow \operatorname{cosec}^2 x\,dx=-du$.

When $x=\dfrac{\pi}{4}\Rightarrow u=1$; when $x=\dfrac{\pi}{2}\Rightarrow u=0$.

$$\therefore I=-\int_1^0 u^2(1+u^2)^2\,du=\int_0^1 u^2(1+u^2)^2\,du$$

$$\Rightarrow I=\int_0^1 (u^6+2u^4+u^2)\,du$$

$$\Rightarrow I=\left[\frac{u^3}{3}+\frac{u^7}{7}+\frac{2u^5}{5}\right]_0^1$$

$$\Rightarrow I=\left[\left(\frac{1}{3}+\frac{1}{7}+\frac{2}{5}\right)-(0)\right]=\frac{92}{105}.$$

29. Rewriting the D.E., we get $\dfrac{dy}{dx}+\dfrac{y}{\sqrt{1-x^2}}=\dfrac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}}$

Comparing the D.E. with $\dfrac{dy}{dx}+yP(x)=Q(x)$, we get $P(x)=\dfrac{1}{\sqrt{1-x^2}}$, $Q(x)=\dfrac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}}$.

Now, the integrating factor $=e^{\int P(x)dx}=e^{\int\frac{1}{\sqrt{1-x^2}}dx}=e^{\sin^{-1}x}$.

So, the solution is given as $y\,e^{\sin^{-1}x}=\displaystyle\int\frac{e^{-\sin^{-1}x}}{\sqrt{1-x^2}}\times e^{\sin^{-1}x}dx+C$

$$\Rightarrow y\,e^{\sin^{-1}x}=\int\frac{1}{\sqrt{1-x^2}}dx+C$$

$\Rightarrow y e^{\sin^{-1} x} = \sin^{-1} x + C$, where C is the constant of integration.

Given that $x = y = 0$, so $0 \times e^{\sin^{-1} 0} = \sin^{-1} 0 + C \quad \Rightarrow C = 0$.

Hence, the particular solution is $y e^{\sin^{-1} x} = \sin^{-1} x$.

OR

We have $\left(x^2 y + yx\sqrt{y^2 - x^2}\right)dx - x^3 dy = 0$

Rewriting the D.E., we get $\dfrac{dy}{dx} = \dfrac{x^2 y + yx\sqrt{y^2 - x^2}}{x^3}$ i.e., $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{y}{x^2}\sqrt{y^2 - x^2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{y}{x}\sqrt{\frac{y^2}{x^2} - 1}$$

Put $y = vx \quad \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dy}{dx}$

$$\therefore v + x\frac{dy}{dx} = v + v\sqrt{v^2 - 1}$$

$$\Rightarrow x\frac{dv}{dx} = v\sqrt{v^2 - 1}$$

$$\Rightarrow \int \frac{dv}{v\sqrt{v^2 - 1}} = \int \frac{dx}{x}$$

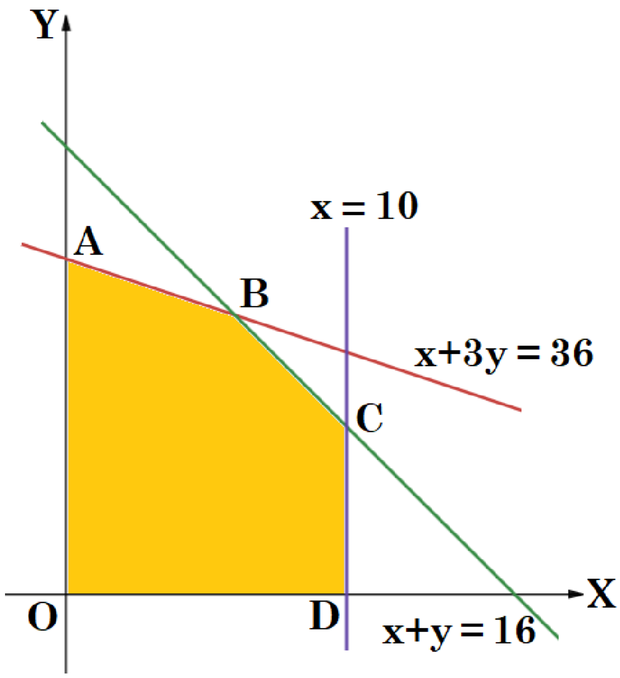
$$\Rightarrow \sec^{-1} v = \log|x| + C$$

$\therefore \sec^{-1}\left(\dfrac{y}{x}\right) = \log|x| + C$, where C is the constant of integration.

30. Consider the graphs shown below.

Corner Points	$Z = 12x + 40y$
A(0, 12)	$480 \leftarrow$ Max.
B(6, 10)	472
C(10, 6)	360
D(10, 0)	120

Clearly, the maximum value of Z is 480 and it is obtained at (0, 12).



31. Let E, F and G be the event of taking out green marbles in the first, second and third draws respectively.

So, $P(E) = P$ (green marble in first draw) $= \dfrac{8}{14}$,

$P(F|E) = P$ (green marble in the second draw) $= \frac{7}{13}$,

$P(G|EF) = P$ (green marble in third draw) $= \frac{6}{12}$.

Required Probability $= P(E) \times P(F|E) \times P(G|EF) = \frac{8}{14} \times \frac{7}{13} \times \frac{6}{12} = \frac{2}{13}$.

OR

Let A : sum of both numbers on two dice was 4, B : the numbers which come up were different.

$$\therefore A = \{(1,3),(2,2),(3,1)\},\ B = \begin{Bmatrix} (1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(1,6),(6,1),(6,2),(6,3),(6,4),(6,5) \end{Bmatrix}$$

$\Rightarrow A \cap B = \{(1,3),(3,1)\}$.

$$\text{Now } P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15}.$$

SECTION D

32. Let the number of litres of the orange juice, beetroot juice and kiwi juice be respectively x, y and z.

Then we get the linear equations :
$500x + 20y + 800z = 1860$, $2x + 5y + 3z = 22$, $100x + 120y + 200z = 760$.

The equations can be written using matrices as, $\begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$

$$\text{where } A = \begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}.$$

Now $|A| = 500(1000 - 360) - 20(400 - 300) + 800(240 - 500) = 110000 \neq 0$, hence matrix A is a non-singular matrix; which means that the system of equations has a unique solution.

Also, $\text{adj}.A = \begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$ $\quad \therefore A^{-1} = \frac{\text{adj}.A}{|A|} = \frac{1}{5500}\begin{bmatrix} 32 & 4600 & -197 \\ -5 & 1000 & 5 \\ -13 & -2900 & 123 \end{bmatrix}$

Since $AX = B \Rightarrow X = A^{-1}B$

$$\text{So, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5500}\begin{bmatrix} 32 & 4600 & -197 \\ -5 & 1000 & 5 \\ -13 & -2900 & 123 \end{bmatrix}\begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

By equality of matrices, we get : $x=2$, $y=3$, $z=1$.
Hence, 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.

33. For the ellipse $\frac{x^2}{81}+\frac{y^2}{36}=1$, expressing y in terms of x as $y=\pm\frac{6}{9}\sqrt{9^2-x^2}$.

Required shaded area $=\left|\int_{-9}^{0}\frac{6}{9}\sqrt{9^2-x^2}\,dx\right|+$ Area of 2 triangles $+\int_{0}^{9}\frac{6}{9}\sqrt{9^2-x^2}\,dx$

$=2\int_{0}^{9}\frac{6}{9}\sqrt{9^2-x^2}\,dx+$ Area of 2 triangles

$=2\times\frac{6}{9}\left[\frac{x}{2}\sqrt{81-x^2}+\frac{81}{2}\sin^{-1}\frac{x}{9}\right]_0^9+2\times\frac{1}{2}\times9\times6$

$=2\times\frac{6}{9}\left\{\left[\frac{9}{2}\sqrt{81-81}+\frac{81}{2}\sin^{-1}\frac{9}{9}\right]-\left[\frac{0}{2}\sqrt{81-0}+\frac{81}{2}\sin^{-1}\frac{0}{9}\right]\right\}+54$

$=2\times\frac{6}{9}\times\frac{81}{2}\times\frac{\pi}{2}+54$

$=27\pi+54$

$=27(\pi+2)$ Sq. units.

34. Comparing $\vec{r}_1=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}_2=\vec{a}_2+\mu\vec{b}_2$ with the given equations to get

$\vec{a}_1=\hat{i}-2\hat{j}+2\hat{k}$, $\vec{a}_2=2\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}_1=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}_2=-\hat{i}+\hat{j}+2\hat{k}$.

Here $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-3\\-1&1&2\end{vmatrix}=7\hat{i}+\hat{j}+3\hat{k}$, and $(\vec{a}_2-\vec{a}_1)=\hat{i}+\hat{k}$.

Now the required shortest distance, $d=\frac{\left|(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$\Rightarrow d=\frac{\left|(7\hat{i}+\hat{j}+3\hat{k}).(\hat{i}+\hat{k})\right|}{\sqrt{49+1+9}}=\frac{|7+0+3|}{\sqrt{59}}=\frac{10}{\sqrt{59}}$ units.

OR

Let L : $\frac{x-4}{1}=\frac{y-2}{3}=\frac{z-1}{2}=\lambda$.
Suppose the random point on the line L be $P(x, y, z)$, which is to be the point of intersection of the two lines. Then $x=\lambda+4$, $y=3\lambda+2$ and $z=2\lambda+1$.
Tara's position is at $(2\hat{i}-2\hat{j}+\hat{k})$ i.e., $T(2,-2,1)$ *say*.
The direction ratios of TP are $\lambda+2,\ 3\lambda+4,\ 2\lambda$.
Using $a_1a_2+b_1b_2+c_1c_2=0$, we get $1(\lambda+2)+3(3\lambda+4)+2(2\lambda)=0$
$\Rightarrow\lambda=-1$.
Hence, $P(3,-1,-1)$.
Therefore, $TP=\sqrt{(3-2)^2+(-1+2)^2+(-1-1)^2}=\sqrt{6}$ units.

35. Let $I=\int x^7\sin(2x^4)\,dx=\int x^4x^3\sin(2x^4)\,dx$

Put $x^4 = u \Rightarrow x^3 dx = \frac{1}{4} du$.

So, $I = \frac{1}{4}\int u \sin(2u) du$

$\Rightarrow I = \frac{1}{4}\left[u\int \sin(2u)\,du - \int\left\{\frac{d}{du}(u)\int \sin(2u)\,du\right\}du\right]$

$\Rightarrow I = \frac{1}{4}\left[-\frac{u}{2}\cos(2u) + \frac{1}{2}\int \cos(2u)\,du\right]$

$\Rightarrow I = \frac{1}{4}\left[-\frac{u}{2}\cos(2u) + \frac{1}{2}\times\frac{1}{2}\sin(2u)\right] + C$

$\therefore I = -\frac{1}{8}x^4\cos(2x^4) + \frac{1}{16}\sin(2x^4) + C$.

OR

Let $I = \int \frac{1}{8-x^3}dx$

$\Rightarrow I = \int \frac{1}{(2-x)(x^2+2x+4)}dx$

Consider $\frac{1}{(2-x)(x^2+2x+4)} = \frac{A}{2-x} + \frac{B(2x+2)+C}{x^2+2x+4}$

$\Rightarrow 1 = A(x^2+2x+4) + B(2x+2)(2-x) + C(2-x)$

$\therefore A = \frac{1}{12}, B = \frac{1}{24}, C = \frac{1}{4}$

Now $I = \int\left[\frac{1}{12}\times\frac{1}{2-x} + \frac{1}{24}\times\frac{(2x+2)}{x^2+2x+4} + \frac{1}{4}\times\frac{1}{x^2+2x+4}\right]dx$

$\Rightarrow I = \int\left[\frac{1}{12}\times\frac{1}{2-x} + \frac{1}{24}\times\frac{(2x+2)}{x^2+2x+4} + \frac{1}{4}\times\frac{1}{(x+1)^2+(\sqrt{3})^2}\right]dx$

$\therefore I = -\frac{1}{12}\times\log|2-x| + \frac{1}{24}\times\log|x^2+2x+4| + \frac{1}{4}\times\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + C$.

SECTION E

36. **(i)** The relation R is, $R = \left\{\begin{matrix}(C, PB), (PB, C), (V, PB), (PB, V), (PB, SwD), (SwD, PB), \\ (PB, ShD), (ShD, PB), (SwD, ShD), (ShD, SwD)\end{matrix}\right\}$.

(ii) For every $(x_1, x_2) \in R$ we have $(x_2, x_1) \in R$; as every direct ship / direct ferry runs in both the directions. So, R is symmetric relation.

(iii) Since $(C, PB) \in R$ as there is a direct ship from Chennai to Port Blair,
Also $(PB, SwD) \in R$ as there is a direct ferry from Port Blair to Swaraj Dweep.
But $(C, SwD) \notin R$ as there is no direction ship/ferry from Chennai to Swaraj Dweep.
Hence, R is not transitive.

OR

(iii) As no two elements of set Y are mapped to a common element in set X so, the function f is one-one.
Also $C \in X$ (codomain of f) but it has no pre-image in Y so, the function f is not onto.

37. **(i)** We have $C(t) = -t^3 + 4.5t^2 + 54t,\ 0 \le t \le 10$.

Put $t=6$, $C(6)=-(6)^3+4.5(6)^2+54(6)$

$\Rightarrow C(6)=-216+4.5\times36+54\times6$

$\Rightarrow C(6)=270$

So, the amount of drug in the bloodstream when the effect of the drug is maximum is 270 mg.

(ii) We have $C(t)=-t^3+4.5t^2+54t,\ 0\le t\le 10$.

$\Rightarrow C'(t)=-3t^2+9t+54$

Put $t=5$, $C'(5)=-3(5)^2+9(5)+54$

$\therefore C'(5)=24$ mg/hr .

(iii) We have $C'(t)=-3t^2+9t+54=-3(t+3)(t-6)$.

Note that, in the interval $(3,\ 4)$, the value of $C'(t)>0$.

Therefore, $C(t)$ is strictly increasing in the interval $(3,\ 4)$.

OR

(iii) We have $C'(t)=-3t^2+9t+54=-3(t+3)(t-6)$.

For $C'(t)=0$, $-3(t+3)(t-6)=0 \quad \Rightarrow t=6$ (since t cannot be negative

Now $C''(t)=-6t+9$

Since $C''(6)=-6\times6+9=-27<0$ so, $C(t)$ attains its maximum at $t=6$ hours.

Hence, 6 hours after the drug is administered, C_{max} is attained.

38. **(i)** Let $P(S)$, $P(C)$ and $P(T)$ be the probabilities that a person selected randomly from the staff prefers sugar, coffee and tea respectively.

We have $P(T)=P(C')=1-0.6=0.4$, $P(S|T)=1-0.2=0.8$.

So, the probability that a randomly selected staff prefers a beverage with sugar is

$$P(S)=P(C)\times P(S|C)+P(T)\times P(S|T)=0.6\times0.9+0.4\times0.8=0.86=\frac{86}{100}=\frac{43}{50} \text{ or, } 86\%.$$

(ii) We have $P(\text{without sugar}|\text{ coffee})=1-0.9=0.1$

$\Rightarrow P(\text{tea})=1-0.6=0.4$.

Using Bayes' theorem, the probability that a staff selected at random prefers coffee given that it is without sugar is

$$P(\text{coffee}|\text{without sugar})=\frac{P(\text{coffee})\times P(\text{without sugar}|\text{coffee})}{P(\text{coffee})\times P(\text{without sugar}|\text{coffee})+P(\text{tea})\times P(\text{without sugar}|\text{tea})}$$

$$=\frac{0.6\times0.1}{0.6\times0.1+0.4\times0.2}=\frac{6}{14}=\frac{3}{7}.$$

Detailed Solutions (PTS-20)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. $\cos^{-1}\left[\cos\left(\frac{13\pi}{8}\right)\right]=\cos^{-1}\left[\cos\left(2\pi-\frac{3\pi}{8}\right)\right]=\cos^{-1}\left[\cos\left(\frac{3\pi}{8}\right)\right]=\frac{3\pi}{8}$.

OR

As $n(A)=4$.

No. of reflexive relations defined on set $A=2^{4(4-1)}=2^{12}=4096$.

No. of symmetric relations defined on set $A=2^{\frac{4(4+1)}{2}}=2^{10}=1024$.

22. Given that $h=2r$, $\frac{dV}{dt}=40\text{ cm}^3/\text{s}$.

Also area of circular base, $\pi r^2=1\text{ m}^2=10000\text{ cm}^2$ …(i)

Since $V=\frac{\pi}{3}\times r^2 h$

$\Rightarrow V=\frac{\pi}{3}\times r^2\times 2r=\frac{2\pi}{3}\times r^3$

$\Rightarrow \frac{dV}{dt}=2\pi r^2\times\frac{dr}{dt}$

$\Rightarrow 40=2\times(10000)\times\frac{dr}{dt}$ [By (i)]

$\Rightarrow \frac{dr}{dt}=0.002\text{ cm/s}$.

23. We have $\overrightarrow{CA}=-2\hat{j}-2\hat{k}$, $\overrightarrow{CB}=3\hat{i}-2\hat{j}+2\hat{k}$.

$\therefore\ \cos C=\frac{\overrightarrow{CA}.\overrightarrow{CB}}{|\overrightarrow{CA}||\overrightarrow{CB}|}$

$\Rightarrow \cos C=\frac{(-2\hat{j}-2\hat{k}).(3\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{4+4}\sqrt{9+4+4}}=\frac{4-4}{2\sqrt{2}\sqrt{17}}=0$

$\therefore C=\frac{\pi}{2}$.

OR

The direction ratios of line AB are $1-2,\ -2-3,\ 3+4$ i.e., $-1,\ -5,\ 7$.

The direction ratios of line BC are $3-1,\ 8+2,\ -11-3$ i.e., $2,\ 10,\ -14$.

Note that, $\frac{-1}{2}=\frac{-5}{10}=\frac{7}{-14}$.

That is, the direction ratios of lines AB and BC are proportional, hence AB is parallel to BC.
But point B is common to both AB and BC.
Therefore, A, B and C are collinear points.

24. $\sin y = x\sin(a+y) \quad \Rightarrow x = \dfrac{\sin y}{\sin(a+y)}$

On differentiating w.r.t. y, $\dfrac{dx}{dy} = \dfrac{\sin(a+y)\times\cos y - \sin y\times\cos(a+y)}{\sin^2(a+y)}$

$\Rightarrow \dfrac{dx}{dy} = \dfrac{\sin(a+y-y)}{\sin^2(a+y)} = \dfrac{\sin a}{\sin^2(a+y)}$

$\therefore \dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$.

Now $\dfrac{d^2y}{dx^2} = \dfrac{1}{\sin a}\times 2\sin(a+y)\cos(a+y)\times\dfrac{dy}{dx}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{1}{\sin a}\times\sin 2(a+y)\times\dfrac{dy}{dx}$

$\Rightarrow \sin a\,\dfrac{d^2y}{dx^2} - \sin 2(a+y)\,\dfrac{dy}{dx} = 0$.

25. $\dfrac{\text{Vector projection of } \vec{a} \text{ on } \vec{b}}{\text{Projection of } \vec{b} \text{ on } \vec{a}} = \dfrac{(\vec{a}.\hat{b})\,\hat{b}}{\vec{b}.\hat{a}} = \left\{\dfrac{\vec{a}.\dfrac{\vec{b}}{|\vec{b}|}}{\vec{b}.\dfrac{\vec{a}}{|\vec{a}|}}\right\}\hat{b} = \left\{\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}\times\dfrac{|\vec{a}|}{\vec{b}.\vec{a}}\right\}\hat{b} = \left\{\dfrac{|\vec{a}|}{|\vec{b}|}\right\}\hat{b}$

$\Rightarrow \qquad = \left\{\dfrac{\sqrt{4+1+4}}{\sqrt{9+25+16}}\right\}\dfrac{3\hat{i}-5\hat{j}+4\hat{k}}{\sqrt{9+25+16}} = \dfrac{3(3\hat{i}-5\hat{j}+4\hat{k})}{50}$.

SECTION C

26. Let $I = \displaystyle\int \dfrac{(\cos x)^{\frac{3}{2}} - (\sin x)^{\frac{3}{2}}}{\sqrt{\sin x\cos x}}\,dx$

$\Rightarrow I = \displaystyle\int \dfrac{(\cos x)\sqrt{\cos x} - (\sin x)\sqrt{\sin x}}{\sqrt{\sin x\cos x}}\,dx$

$\Rightarrow I = \displaystyle\int \left[\dfrac{(\cos x)\sqrt{\cos x}}{\sqrt{\sin x\cos x}} - \dfrac{(\sin x)\sqrt{\sin x}}{\sqrt{\sin x\cos x}}\right]dx$

$\Rightarrow I = \displaystyle\int \left[\dfrac{\cos x}{\sqrt{\sin x}} - \dfrac{\sin x}{\sqrt{\cos x}}\right]dx$

$\Rightarrow I = 2\sqrt{\sin x} + 2\sqrt{\cos x} + C \qquad \left[\because \displaystyle\int \dfrac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C\right.$

27. Let E : getting a six on the pair of dice by A.

So, $P(E) = \dfrac{5}{36} \quad \therefore P(\bar{E}) = \dfrac{31}{36}$. $\qquad [\because \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

Also, let F : getting a seven on the pair of dice by B.

So, $P(F)=\frac{6}{36}$ $\therefore P(\bar{F})=\frac{30}{36}=\frac{5}{6}$. $[\because \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

If A starts the game, then he can win in first, third, fifth, … throws.

Hence $P(\text{A wins})=P(E)+P(\bar{E}\bar{F}E)+P(\bar{E}\bar{F}\bar{E}\bar{F}E)+...$

$$\Rightarrow P(\text{A wins})=\frac{5}{36}+\left(\frac{31\times5}{216}\right)\frac{5}{36}+\left(\frac{31\times5}{216}\right)^2\frac{5}{36}+... \quad \left[\because a+ar+ar^2+...\infty=\frac{a}{1-r}\text{; for infinite G.P.}\right.$$

$$\Rightarrow P(\text{A wins})=\frac{\frac{5}{36}}{1-\frac{31\times5}{216}}=\frac{30}{216-155}=\frac{30}{61}$$

and, $P(\text{B wins})=1-P(\text{A wins})=\frac{31}{61}$.

OR

Let A and B denote the events that husband and wife are selected, respectively.

So, $P(A)=\frac{1}{4},\ P(B)=\frac{1}{7},\ P(A')=1-P(A)=\frac{3}{4},\ P(B')=1-P(B)=\frac{6}{7}$.

(i) $P(\text{both of them will be selected})=P(A)\times P(B)=\frac{1}{4}\times\frac{1}{7}=\frac{1}{28}$

(ii) $P(\text{none of them will be selected})=P(A')\times P(B')=\frac{3}{4}\times\frac{6}{7}=\frac{18}{28}$

(iii) $P(\text{at least one of them will be selected})=P(A)\times P(B')+P(A')P(B)+P(A)\times P(B)$

$$=1-P(A')\times P(B')=1-\frac{3}{4}\times\frac{6}{7}=\frac{10}{28}.$$

28. Let $I=\int_0^{\pi}|\cos x-\sin x|dx$

$$\Rightarrow I=\int_0^{\pi/4}|\cos x-\sin x|dx+\int_{\pi/4}^{\pi}|\cos x-\sin x|dx$$

$$\Rightarrow I=\int_0^{\pi/4}(\cos x-\sin x)dx-\int_{\pi/4}^{\pi}(\cos x-\sin x)dx$$

$$\Rightarrow I=[\sin x+\cos x]_0^{\pi/4}-[\sin x+\cos x]_{\pi/4}^{\pi}$$

$$\Rightarrow I=\left[\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right]-[\sin 0+\cos 0]-[\sin\pi+\cos\pi]+\left[\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right]$$

$$\Rightarrow I=\left[\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right]-[0+1]-[0-1]+\left[\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right]$$

$\therefore I=2\sqrt2$.

OR

Let $I=\int_0^{\pi}\frac{x\sin x}{4+\cos^2 x}dx$ …(i)

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{4+\cos^2(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{4+\cos^2 x}dx \qquad \text{…(ii)}$$

Adding (i) and (ii), we get : $2I = \pi\int_0^{\pi} \frac{\sin x}{4+\cos^2 x} dx$

Let $f(x) = \frac{\sin x}{4+\cos^2 x}$

$\Rightarrow f(\pi - x) = \frac{\sin(\pi - x)}{4+\cos^2(\pi - x)} = \frac{\sin x}{4+\cos^2 x} = f(x)$.

By using $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ if $f(2a-x) = f(x)$, we get : $2I = 2\pi\int_0^{\pi/2} \frac{\sin x}{4+\cos^2 x} dx$

$\therefore I = \pi\int_1^0 \frac{-dt}{4+t^2}$

Put $\cos x = t \Rightarrow \sin x\, dx = -dt$

When $x = 0,\ t = 1;$ when $x = \frac{\pi}{2},\ t = 0$

$\Rightarrow I = \pi\int_0^1 \frac{dt}{4+t^2} = \pi \times \frac{1}{2}\left[\tan^{-1}\frac{t}{2}\right]_0^1$

$\Rightarrow I = \frac{\pi}{2}\left(\tan^{-1}\frac{1}{2} - 0\right)$

Hence, $I = \frac{\pi}{2} . \tan^{-1}\left(\frac{1}{2}\right)$.

29. $(dy - dx) + \cos x\,(dy + dx) = 0$

$\Rightarrow \left(\frac{dy}{dx} - 1\right) + \cos x\left(\frac{dy}{dx} + 1\right) = 0$

$\Rightarrow \frac{dy}{dx}(1+\cos x) - 1 + \cos x = 0$

$\Rightarrow \frac{dy}{dx}(1+\cos x) = 1 - \cos x$

$\Rightarrow \int dy = \int \frac{1-\cos x}{1+\cos x} dx$

$\Rightarrow \int dy = \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$

$\Rightarrow \int dy = \int \tan^2 \frac{x}{2} dx$

$\Rightarrow \int dy = \int \left(\sec^2 \frac{x}{2} - 1\right) dx$

$\Rightarrow y = 2\tan\frac{x}{2} - x + C$.

OR

$x\cos\left(\frac{y}{x}\right) \times \frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$$\Rightarrow \frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}+\sec\left(\frac{y}{x}\right)$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.

So, $v + x\frac{dv}{dx} = v + \sec v$

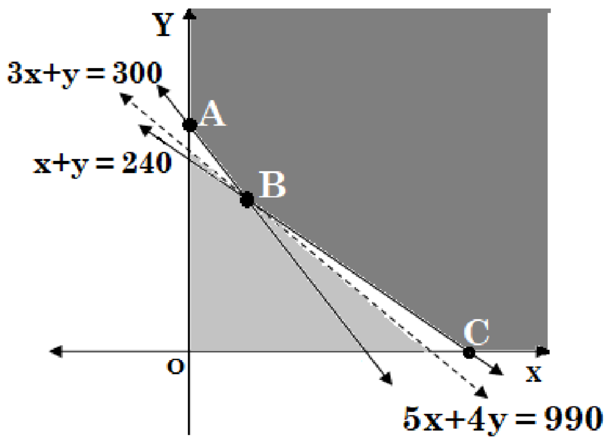
$$\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$$

$$\Rightarrow \sin v = \log x + C$$

$$\therefore \sin\left(\frac{y}{x}\right) = \log x + C.$$

30. Consider the graph shown.

Corner Points	Value of Z
A(0, 300)	2400
B(30, 210)	1980 ← Minimum
C(240, 0)	2400



Since the feasible region is unbounded so 1980 may or may not be the minimum value of Z.

To check, we draw $10x + 8y < 1980$ or, $5x + 4y < 990$.
As $5x + 4y < 990$ has no point in common to the feasible region.
Hence, minimum value of Z is 1980.

31. Let $I = \int \frac{x\ dx}{1 + x\tan x}$

$$\Rightarrow I = \int \frac{x\cos x\ dx}{\cos x + x\sin x}$$

Put $\cos x + x\sin x = t \Rightarrow (-\sin x + x\cos x + \sin x)dx = x\cos x\, dx = dt$.

So, $I = \int \frac{dt}{t}$

$$\Rightarrow I = \log|t| + C$$

$$\therefore I = \log|\cos x + x\sin x| + C.$$

SECTION D

32. Consider $y = x^2 \ldots$(i) and $y = x \ldots$(ii)
On solving (i) and (ii), we get the points of intersection as $(0, 0)$ and $(1, 1)$.

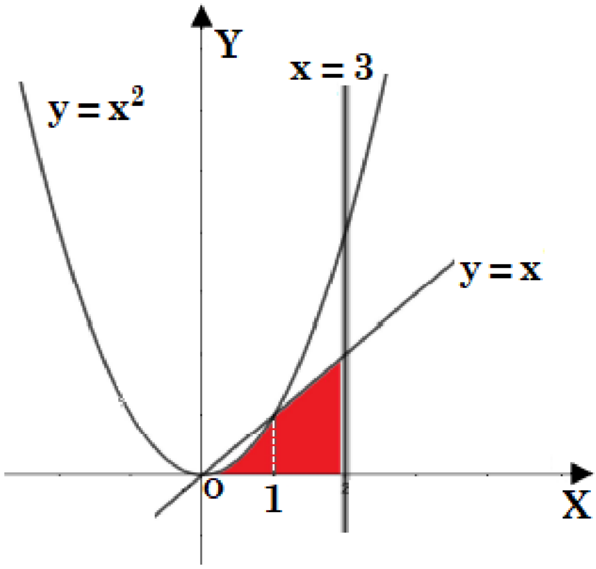
$$\text{Required Area} = \int_0^1 y_{\text{parabola}}\,dx + \int_1^3 y_{\text{line}}\,dx$$

$$\Rightarrow \text{Required area} = \int_0^1 x^2 dx + \int_1^3 x\,dx$$

$$\Rightarrow \text{Required area} = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^3$$

$$\Rightarrow \text{Required area} = \frac{1}{3} + 4$$

$$\Rightarrow \text{Required area} = \frac{13}{3} \text{ Sq. units.}$$



33. Here $Y = \{n^2 : n \in N\} \subset N$.

Also, $f : N \to Y$ as $f(n) = n^2$.

Let $n_1, n_2 \in N$ such that $f(n_1) = f(n_2)$

$\Rightarrow n_1^2 = n_2^2$

$\Rightarrow n_1^2 - n_2^2 = 0$

$\Rightarrow (n_1 - n_2)(n_1 + n_2) = 0$

$\Rightarrow (n_1 - n_2) = 0$ i.e., $n_1 = n_2$ $\qquad$ ($\because n_1 + n_2 \neq 0$ for natural values

That is, f is one-one.

Now let $y \in Y$ such that $y = f(n)$

$\Rightarrow y = n^2$

$\Rightarrow n = \pm\sqrt{y}$

But $f : N \to Y$ so, $n = \sqrt{y}$ as $n \in N$.

That is, for all $y \in Y \ \exists\ n \in N$ such that $y = f(n)$

So, f is onto.

OR

Given that the relation R in the set A of points in a plane given by R = {(P, Q) : Distance of the point P from the origin is same as the distance of point Q from the origin}.

That is, R = {(P, Q) : $OP = OQ$, where O is the origin}.

Reflexivity : Let $P \in A$. Since $OP = OP$ so, $(P, P) \in R$.

$\therefore$ R is reflexive.

Symmetry : Let $P, Q \in A$. Let $(P,Q) \in R$.

So, $OP = OQ \Rightarrow OQ = OP \Rightarrow (Q,P) \in R$

$\therefore$ R is symmetric.

Transitivity : Let $P, Q, S \in A$. Let $(P,Q) \in R$ and $(Q,S) \in R$.

So, $OP = OQ$ and $OQ = OS \Rightarrow OP = OS \Rightarrow (P,S) \in R$

$\therefore$ R is transitive.

Hence, R is equivalence relation.

34. For the lines, $\vec{a}_1 = \hat{i} + \hat{j},\ \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k};\ \vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k},\ \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$.

Therefore, $\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k},\ \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}=3\hat{i}-\hat{j}-7\hat{k}$.

So, $\text{S.D.}=\dfrac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$\Rightarrow \text{S.D.}=\dfrac{\left|(\hat{i}-\hat{k}).(3\hat{i}-\hat{j}-7\hat{k})\right|}{\left|3\hat{i}-\hat{j}-7\hat{k}\right|}$

$\Rightarrow \text{S.D.}=\dfrac{|3+7|}{\sqrt{9+1+49}}=\dfrac{10}{\sqrt{59}}$ units.

Let the acute angle between the lines be θ.

So, $\cos\theta=\dfrac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}=\dfrac{\left|(2\hat{i}-\hat{j}+\hat{k}).(3\hat{i}-5\hat{j}+2\hat{k})\right|}{\sqrt{4+1+1}\sqrt{9+25+4}}$

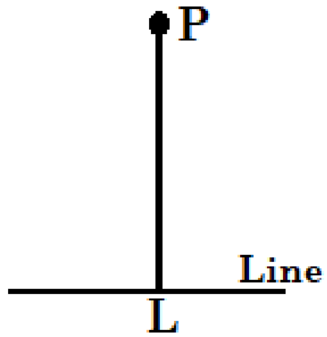
$\Rightarrow \cos\theta=\dfrac{|6+5+2|}{\sqrt{6}\sqrt{38}}$

$\therefore \theta=\cos^{-1}\left(\dfrac{13}{2\sqrt{57}}\right)$.

OR

Equation of the line through $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ and parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ is,

L : $\vec{r}=4\hat{i}+2\hat{j}+2\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$.



Let M be the foot of perpendicular on line L drawn from point P with position vector $\vec{r}_1=\hat{i}+2\hat{j}+3\hat{k}$.

So, position vector of M which lies on line L is $(2\lambda+4)\hat{i}+(3\lambda+2)\hat{j}+(6\lambda+2)\hat{k}$.

$\therefore \overrightarrow{PL}=(2\lambda+3)\hat{i}+3\lambda\hat{j}+(6\lambda-1)\hat{k}$

Since $\overrightarrow{PL}$ shall be perpendicular to line L so, $\overrightarrow{PL}.\vec{b}=0$.

That is, $\{(2\lambda+3)\hat{i}+3\lambda\hat{j}+(6\lambda-1)\hat{k}\}.(2\hat{i}+3\hat{j}+6\hat{k})=0$

$\Rightarrow \lambda=0$

$\therefore \overrightarrow{PL}=3\hat{i}-\hat{k}$.

Therefore the length of the perpendicular is, $\left|\overrightarrow{PL}\right|=\sqrt{9+1}=\sqrt{10}$ units.

35. $x+y+z=21,\ 4x+3y+2z=60,\ 6x+2y+3z=70$.

Since $AX=B$ where $A=\begin{pmatrix}1 & 1 & 1\\ 4 & 3 & 2\\ 6 & 2 & 3\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ and $B=\begin{pmatrix}21\\ 60\\ 70\end{pmatrix}$.

Now $|A|=1(5)-1(0)+1(-10)=-5\neq 0\ \therefore A^{-1}$ exists.

Consider A_{ij} be the cofactor of corresponding element a_{ij}.

$A_{11}=5,\quad A_{21}=-1,\quad A_{31}=-1,$

$A_{12}=0,\quad A_{22}=-3,\quad A_{32}=2,$

$A_{13}=-10,\quad A_{23}=4,\quad A_{33}=-1$

$$\therefore A^{-1}=\frac{1}{|A|}.[\text{adj.}(A)]=\frac{1}{-5}\begin{bmatrix}5&-1&-1\\0&-3&2\\-10&4&-1\end{bmatrix}$$

As $X=A^{-1}B \Rightarrow \begin{pmatrix}x\\y\\x\end{pmatrix}=\frac{1}{-5}\begin{pmatrix}5&-1&-1\\0&-3&2\\-10&4&-1\end{pmatrix}\begin{pmatrix}21\\60\\70\end{pmatrix}=\begin{pmatrix}5\\8\\8\end{pmatrix}$

By equality of matrices, we get $x=5,\ y=8,\ z=8$.

SECTION E

36. **(i)** $P(A_1)=\frac{4}{10},\ P(A_2)=\frac{4}{10},\ P(A_3)=\frac{2}{10}$.

(ii) As $P(E\,|\,A_1)=\frac{45}{100},\ P(E\,|\,A_2)=\frac{60}{100},\ P(E\,|\,A_3)=\frac{35}{100}$

So, $P(E\,|\,A_1)+P(E\,|\,A_2)+P(E\,|\,A_3)=\frac{45}{100}+\frac{60}{100}+\frac{35}{100}$

$=\frac{140}{100}=1.4$.

(iii) $P(E)=P(A_1)P(E\,|\,A_1)+P(A_2)P(E\,|\,A_2)+P(A_3)P(E\,|\,A_3)$

$\Rightarrow P(E)=\frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}$

$\Rightarrow P(E)=\frac{49}{100}=0.49$ or 49%.

OR

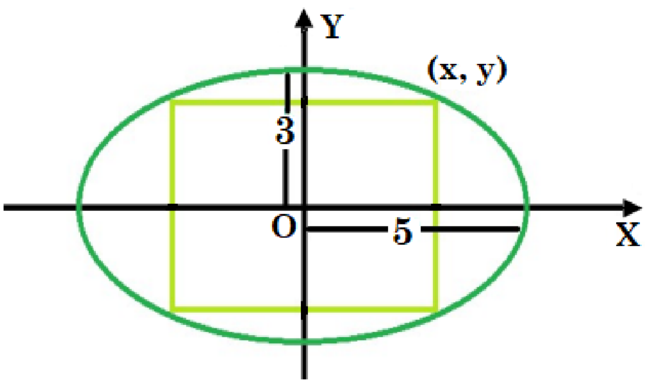
(iii) As $E:$ seed germinates so, $\overline{E}:$ seed does not germinate.

By using Bayes' theorem, $P(A_2\,|\,\overline{E})=\dfrac{P(A_2)P(\overline{E}\,|\,A_2)}{P(A_1)P(\overline{E}\,|\,A_1)+P(A_2)P(\overline{E}\,|\,A_2)+P(A_3)P(\overline{E}\,|\,A_3)}$

$$\Rightarrow P(A_2\,|\,\overline{E})=\frac{\frac{4}{10}\times\frac{40}{100}}{\frac{4}{10}\times\frac{55}{100}+\frac{4}{10}\times\frac{40}{100}+\frac{2}{10}\times\frac{65}{100}}$$

$\Rightarrow P(A_2\,|\,\overline{E})=\frac{16}{51}$.

37. **(i)** Let $(x,\,y)=\left(x,\,\frac{3}{5}\sqrt{25-x^2}\right)$ be the upper right vertex of the rectangle.



The area function, $A = 2x \times 2\left(\frac{3}{5}\sqrt{25-x^2}\right)$

$\Rightarrow A = \frac{12}{5}x\sqrt{25-x^2}$, $x \in (0, 5)$.

(ii) $\frac{dA}{dx} = \frac{12}{5}\left[x \times \frac{-x}{\sqrt{25-x^2}} + \sqrt{25-x^2}\right]$

$\Rightarrow \frac{dA}{dx} = \frac{12}{5} \times \frac{25-2x^2}{\sqrt{25-x^2}} = -\frac{12}{5} \times \frac{(\sqrt{2}x+5)(\sqrt{2}x-5)}{\sqrt{25-x^2}}$

For $\frac{dA}{dx} = 0 \quad \Rightarrow x = \frac{5}{\sqrt{2}}$

So, $x = \frac{5}{\sqrt{2}}$ is the critical point.

(iii) For the values of x less than $\frac{5}{\sqrt{2}}$ and close to $\frac{5}{\sqrt{2}}$, $\frac{dA}{dx} > 0$ and for the values of x greater than $\frac{5}{\sqrt{2}}$ and close to $\frac{5}{\sqrt{2}}$, $\frac{dA}{dx} < 0$.

Hence, by the first derivative test, there is a local maximum at the critical point $x = \frac{5}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x = \frac{5}{\sqrt{2}}$.

$\therefore$ For maximum area of the soccer field, its length should be $5\sqrt{2}$ units and its width should be $3\sqrt{2}$ units.

OR

(iii) $A = 2x \times 2\left(\frac{3}{5}\sqrt{25-x^2}\right)$, $x \in (0, 5)$.

Squaring both sides, we get $Z = A^2 = \frac{16 \times 9}{25}x^2(25-x^2) = \frac{144}{25}(25x^2 - x^4)$, $x \in (0, 5)$.

($\because$ A is maximum when Z is maximum.)

Now $\frac{dZ}{dx} = \frac{144}{25}(50x - 4x^3) = \frac{288}{25}x(5+\sqrt{2}x)(5-\sqrt{2}x)$

For $\frac{dZ}{dx} = 0 \quad \Rightarrow x = \frac{5}{\sqrt{2}}$

Also, $\frac{d^2Z}{dx^2} = \frac{288}{25}(25-6x^2)$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{5}{\sqrt{2}}} = \frac{288}{25}(25-75) = -576 < 0$.

Therefore, by the second derivative test there is a local maximum value of Z at the critical point $x = \frac{5}{\sqrt{2}}$.

Since there is only one critical point therefore, Z is maximum at $x = \frac{5}{\sqrt{2}}$.

Hence, A is maximum at $x = \frac{5}{\sqrt{2}}$.

$\therefore$ For maximum area of the soccer field, its length should be $5\sqrt{2}$ units and its width should be $3\sqrt{2}$ units.

38. **(i)** Volume of the cylindrical tank, $V = \pi r^2 h$.

At $r = 10$ m, $V = 100\pi h$

$\therefore \frac{dV}{dt} = 100\pi \times \frac{dh}{dt}$

$\Rightarrow 314 = 100\pi \times \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{314}{100 \times 3.14} = 1$ metre/hour .

(ii) Curved surface area of the tank is, $A = 2\pi r h$

$\Rightarrow A = 2\pi \times 10 \times h$

$\Rightarrow \frac{dA}{dt} = 20\pi \times \frac{dh}{dt}$

$\therefore \frac{dA}{dt} = 20\pi \times 1 = 20 \times 3.14 = 62.8\ m^2/\text{hour}$.

Detailed Solutions (PTS-21)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. Given that $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\dfrac{3\pi}{2}$

Clearly, $\sin^{-1}\alpha=\dfrac{\pi}{2}$, $\sin^{-1}\beta=\dfrac{\pi}{2}$ and $\sin^{-1}\gamma=\dfrac{\pi}{2}$ (as maximum value of $\sin^{-1}$ is $\dfrac{\pi}{2}$

$\Rightarrow \alpha=\sin\dfrac{\pi}{2}$, $\beta=\sin\dfrac{\pi}{2}$ and $\gamma=\sin\dfrac{\pi}{2}$

$\therefore \alpha=\beta=\gamma=1$.

Now $\left[\alpha(\beta+\gamma)-\beta(\gamma+\alpha)+\gamma(\alpha+\beta)\right]^{(\alpha+\beta+\gamma)}=\left[(1)(1+1)-(1)(1+1)+(1)(1+1)\right]^{1+1+1}=(2-2+2)^3=8$.

OR

$$\cot^{-1}\left\{\frac{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}+\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}-\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\right\}$$

$\left[\because \dfrac{\pi}{2}<x<\pi \quad \therefore \dfrac{\pi}{4}<\dfrac{x}{2}<\dfrac{\pi}{2}\right.$ Also, $\sin\dfrac{x}{2}>\cos\dfrac{x}{2}$

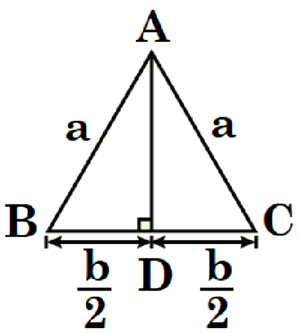
$$=\cot^{-1}\left\{\frac{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|-\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}\right\}=\cot^{-1}\left\{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}\right\}$$

$$=\cos^{-1}\left(\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right)=\cot^{-1}\left(\tan\frac{x}{2}\right)$$

$\left[\because \dfrac{\pi}{4}<\dfrac{x}{2}<\dfrac{\pi}{2} \quad \therefore 0<\dfrac{\pi}{2}-\dfrac{x}{2}<\dfrac{\pi}{4}\right.$

$$=\cot^{-1}\left(\cot\left(\frac{\pi}{2}-\frac{\pi}{2}\right)\right)=\frac{\pi}{2}-\frac{x}{2}.$$

22. In isosceles ΔABC, let $AB=AC=a$ and $BC=b$ (given).



Given $\dfrac{da}{dt}=-3$ cm/sec.

In triangle ADB, $AD=\sqrt{a^2-\dfrac{b^2}{4}}$

Now $A=\text{Area of }\Delta ABC=\dfrac{1}{2}(BC)(AD)$

$$\Rightarrow A=\left(\frac{1}{2}\right)(b)\sqrt{a^2-\frac{b^2}{4}}$$

$$\Rightarrow \frac{dA}{dt}=\left(\frac{b}{2}\right)\left(\frac{1}{2}\right)\left(a^2-\frac{b^2}{4}\right)^{-1/2}\times\left(2a\frac{da}{dt}\right)$$

$$\Rightarrow \frac{dA}{dt}=\frac{b(2a)(-3)}{4\sqrt{a^2-\frac{b^2}{4}}}=\frac{-3ab}{2\sqrt{a^2-\frac{b^2}{4}}}$$

$$\Rightarrow \left(\frac{dA}{dt}\right)_{\text{at } a=b}=\frac{-3\times b\times b}{2\sqrt{b^2-\frac{b^2}{4}}}=\frac{-3b^2}{\frac{2\sqrt{3}\,b}{2}}=-\sqrt{3}\,b$$

$\therefore$ Area is decreasing at the rate of $\sqrt{3}\,b\ \text{cm}^2/\text{sec}$.

OR

Let the side of a cube be x unit.

Volume of cube $=V=x^3$

$$\Rightarrow \frac{dV}{dt}=3x^2\frac{dx}{dt}=k\ \text{(constant)}$$

$$\Rightarrow \frac{dx}{dt}=\frac{k}{3x^2}\quad \ldots(i)$$

Surface area $=S=6x^2$

$$\Rightarrow \frac{dS}{dt}=12x\frac{dx}{dt}$$

$$\Rightarrow \frac{dS}{dt}=12x\times\frac{k}{3x^2}=4\left(\frac{k}{x}\right)\qquad \text{[By (i)}$$

$$\therefore \frac{dS}{dt}\propto\frac{1}{x}.$$

Hence, the surface area of the cube varies inversely as length of side.

23. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

Now $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=(-\vec{c}).(-\vec{c})$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=\vec{c}.\vec{c}$

$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2=|\vec{c}|^2$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2$

$\Rightarrow 2(3)(5)\cos\theta=49-9-25$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}$

$\therefore \theta=\frac{\pi}{3}.$

24. Let $P(3\lambda-2,\ 2\lambda-1,\ 2\lambda+3)$ be any point on a line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ which is at a distance of 5 units from the point $Q(1,\ 3,\ 3)$.

According to the question, $PQ=5$

$\Rightarrow (PQ)^2 = 25$

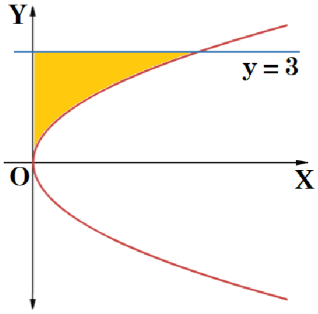
$\Rightarrow (3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2 = 25$

$\Rightarrow 17\lambda^2 - 34\lambda = 0$

$\Rightarrow \lambda = 0$ or 2

Required Point is $(-2, -1, 3)$ or $(4, 3, 7)$.

25. The curves are $y^2 = 4x$, $x = 0$ and line $y = 3$.



$$\text{Required area} = \int_0^3 \frac{y^2}{4} dy = \frac{1}{4} \times \left[\frac{y^3}{3}\right]_0^3$$

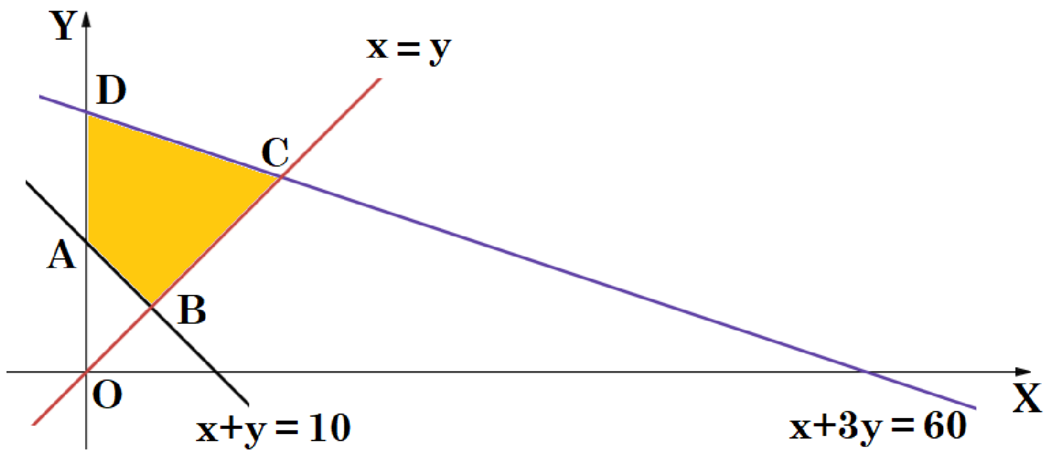
$$= \frac{27}{12} - 0$$

$$= \frac{9}{4} \text{ Square units.}$$

SECTION C

26. Consider the graph shown (on next page).

Corner Points	Value of Z
A(0, 10)	$Z = 0 + 90 = 90$
B(5, 5)	$Z = 15 + 45 = 60 \leftarrow$ Minimum
C(15, 15)	$Z = 45 + 135 = 180$
D(0, 20)	$Z = 0 + 180 = 180$



Hence, the minimum value of $Z = 60$.

27. Let E : sum of numbers is 8, F : the red die shows a number less than 4.

$$\therefore E = \{(2,6),(6,2),(3,5),(5,3),(4,4)\},\ F = \left\{\begin{array}{l}(1,1),(2,1),\ldots,(6,1),(1,2),\\(2,2),\ldots,(6,2),(1,3),(2,3),\ldots,(6,3)\end{array}\right\}$$

$\Rightarrow E \cap F = \{(6,2),(5,3)\}$

$$\text{Therefore, } P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}}$$

$$\therefore P(E \mid F) = \frac{2}{18} = \frac{1}{9}.$$

OR

Let $P(A)=x$ and $P(B)=y$.

According to the question, $P(A\cap B)=\frac{1}{6}$ and $P(A'\cap B')=\frac{1}{3}$

$\Rightarrow P(A)\times P(B)=\frac{1}{6}$ and $P(A')\times P(B')=\frac{1}{3}$

$\Rightarrow xy=\frac{1}{6}$ and $(1-x)(1-y)=\frac{1}{3}$

Now $1-x-y+xy=\frac{1}{3}$ $\qquad \Rightarrow 1-\frac{1}{3}+\frac{1}{6}=x+y$ $\qquad$ (Since $xy=\frac{1}{6}$

$\Rightarrow \frac{5}{6}=x+\frac{1}{6x}$ i.e., $\frac{5}{6}=\frac{6x^2+1}{6x}$ i.e., $6x^2-5x+1=0$ i.e., $(2x-1)(3x-1)=0$

Clearly, we have $x=P(A)=\frac{1}{2}$ or, $\frac{1}{3}$.

28. Let $I=\int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\frac{\cos x}{(\sin x-1)(\sin x-2)}dx$

Put $\sin x=t \quad \Rightarrow \cos x\,dx=dt$

$\therefore I=\int\frac{dt}{(t-1)(t-2)}=\int\left(\frac{-1}{t-1}+\frac{1}{t-2}\right)dt$

$\Rightarrow I=-\log|t-1|+\log|t-2|$

$\Rightarrow I=\log\left|\frac{t-2}{t-1}\right|+c$

$\Rightarrow I=\log\left|\frac{\sin x-2}{\sin x-1}\right|+c$.

OR

Let $I=\int\frac{\sin\phi}{\sqrt{\sin^2\phi+2\cos\phi+3}}d\phi$

$\Rightarrow I=\int\frac{\sin\phi}{\sqrt{1-\cos^2\phi+2\cos\phi+3}}d\phi$

$\Rightarrow I=\int\frac{\sin\phi\,d\phi}{\sqrt{4+2\cos\phi-\cos^2\phi}}$

Put $\cos\phi=t \Rightarrow \sin\phi\,d\phi=-dt$

$\therefore I=\int\frac{-dt}{\sqrt{4+2t-t^2}}=-\int\frac{dt}{\sqrt{-\left[t^2-2t-4\right]}}$

$\Rightarrow I=-\int\frac{dt}{\sqrt{-\left[t^2-2t+1-5\right]}}=-\int\frac{dt}{\sqrt{\left(\sqrt{5}\right)^2-(t-1)^2}}$

$\Rightarrow I=-\sin^{-1}\left(\frac{t-1}{\sqrt{5}}\right)+c$

$\Rightarrow I=-\sin^{-1}\left(\frac{\cos\phi-1}{\sqrt{5}}\right)+c$.

29. We have $2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$

$\Rightarrow 2ye^{\frac{x}{y}}dx=\left(2xe^{\frac{x}{y}}-y\right)dy$

$\Rightarrow \dfrac{dx}{dy}=\left(\dfrac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}\right)$

Put $x=vy \quad \Rightarrow \dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$

Now $v+y\dfrac{dv}{dy}=\dfrac{2vye^{v}-y}{2ye^{v}}$

$\Rightarrow y\dfrac{dv}{dy}=\dfrac{2vye^{v}-y-2vye^{v}}{2ye^{v}}$

$\Rightarrow \dfrac{dv}{dy}=\dfrac{-1}{2ye^{v}}$

$\Rightarrow \int 2e^{v}dv=\int -\dfrac{1}{y}dy$

$\Rightarrow 2e^{v}=-\log y+c$

Hence, $2e^{\frac{x}{y}}+\log y=c$.

OR

We have $\dfrac{dy}{dx}-3y\cot x=\sin 2x$

On comparing with $\dfrac{dy}{dx}+Py=Q$, we get $P=-3\cot x$, $Q=\sin 2x$

Now $\text{I.F.}=e^{\int P\,dx}=e^{\int -3\cot x\,dx}=e^{-3\log\sin x}=e^{\log(\sin x)^{-3}}=(\sin x)^{-3}=\dfrac{1}{\sin^{3}x}$

The solution of given differential equation is $y(\text{I.F.})=\int Q(\text{I.F.})\,dx+c$

$\Rightarrow y\left(\dfrac{1}{\sin^{3}x}\right)=\int \sin 2x\times\dfrac{1}{\sin^{3}x}dx+c$

$\Rightarrow \dfrac{y}{\sin^{3}x}=\int 2\sin x\cos x\times\dfrac{1}{\sin^{3}x}dx+c$

$\Rightarrow \dfrac{y}{\sin^{3}x}=\int 2\cot x\,\text{cosec}\,x\,dx+c$

$\Rightarrow \dfrac{y}{\sin^{3}x}=-2\,\text{cosec}\,x+c$

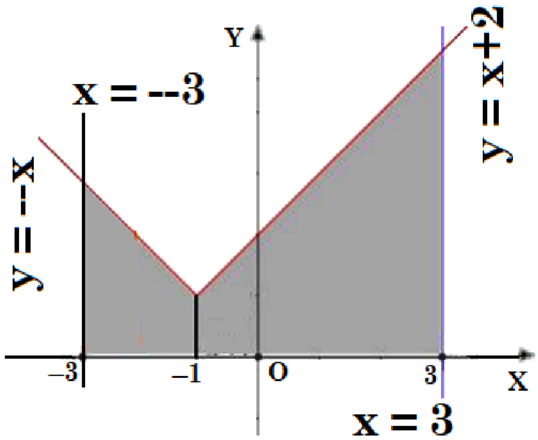
$\Rightarrow \dfrac{y}{\sin^{3}x}=-2\times\dfrac{1}{\sin x}+c$

$\therefore y=-2\sin^{2}x+c\sin^{3}x$.

30. We have $y=1+|x+1|$, $x=-3$, $x=3$, $y=0$.

So, $y=\begin{cases}1+(-x-1)=-x, \text{ if } -3\le x<-1\\ 1+(x+1)=x+2, \text{ if } 1\le x\le 3\end{cases}$.

Consider the adjacent diagram shown.



$$\text{Required Area } =\int_{-3}^{-1}-x\,dx+\int_{-1}^{3}(x+2)\,dx$$

$$=\left[\frac{-x^2}{2}\right]_{-3}^{-1}+\left[\frac{x^2}{2}+2x\right]_{-1}^{3}$$

$$=\frac{-1}{2}(1-9)+\left[\left(\frac{9}{2}+6\right)-\left(\frac{1}{2}-2\right)\right]$$

$$=4+12$$

$$=16 \text{ Square units}.$$

31. $x=a\sin t-b\cos t,\ y=a\cos t+b\sin t$

So, $\dfrac{dx}{dt}=a\cos t+b\sin t=y$ and, $\dfrac{dy}{dt}=-a\sin t+b\cos t=-x$

As $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\qquad\therefore \dfrac{dy}{dx}=\dfrac{-x}{y}$

Now $\dfrac{d^2y}{dx^2}=\dfrac{y(-1)-(x)\times\frac{dy}{dx}}{(y)^2}$

$\Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{-y+x\left(-\frac{x}{y}\right)}{(y)^2}$

$\Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{-y^2-x^2}{y^3}$

$\therefore \quad \dfrac{d^2y}{dx^2}=-\dfrac{(x^2+y^2)}{y^3}$.

SECTION D

32. For $A=\begin{bmatrix}1&2&3\\2&3&-3\\-3&2&-4\end{bmatrix}$, $|A|=1(-12+6)-2(-8-9)+3(4+9)=-6+34+39=67$

Also, $\text{Adj. } A=\begin{bmatrix}-6&14&-15\\17&5&9\\13&-8&-1\end{bmatrix}$

Therefore, $A^{-1}=\dfrac{1}{67}\begin{bmatrix}-6&14&-15\\17&5&9\\13&-8&-1\end{bmatrix}$.

Now the equations are $x+2y-3z=-4;\ 2x+3y+2z=14;\ 3x-3y-4z=-15$.

The matrix form of the equation is $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 14 \\ -15 \end{bmatrix}$.

Note that, $A^T X = B$

$\Rightarrow X = (A^T)^{-1} B = (A^{-1})^T B$

$\Rightarrow X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 14 \\ -15 \end{bmatrix}$

$\Rightarrow X = \frac{1}{67}\begin{bmatrix} 24+238-195 \\ -56+70+120 \\ 60+126+15 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 67 \\ 134 \\ 201 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

By equality of matrices, we get $x = 1,\ y = 2,\ z = 3$.

33. Let $P(2\lambda+3, \lambda+3, \lambda)$ be any random point on line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = \lambda$.

Let the line through origin and making an angle of $\frac{\pi}{3}$ with the given line be along OP.

Then direction ratios of OP are proportional to $2\lambda+3-0, \lambda+3-0, \lambda-0$ i.e., $2\lambda+3, \lambda+3, \lambda$.

Also, direction ratios of the given line are proportional to $2, 1, 1$.

$\therefore \cos\frac{\pi}{3} = \frac{(2\lambda+3)(2)+(\lambda+3)(1)+(\lambda)(1)}{\sqrt{(2\lambda+3)^2+(\lambda+3)^2+(\lambda)^2}\sqrt{2^2+1^2+1^2}}$

$\Rightarrow \frac{1}{2} = \frac{6\lambda+9}{\sqrt{6\lambda^2+18\lambda+18}\sqrt{6}}$

$\Rightarrow \frac{1}{2} = \frac{3(2\lambda+3)}{6\sqrt{\lambda^2+3\lambda+3}}$

$\Rightarrow \sqrt{\lambda^2+3\lambda+3} = (2\lambda+3)$

Squaring both sides, we get $\lambda^2+3\lambda+3 = (2\lambda+3)^2$

$\Rightarrow \lambda^2+3\lambda+2 = 0$

$\Rightarrow \lambda = -1, -2$

Therefore, the coordinates of points are $(1, 2, -1)$ or $(-1, 1, -2)$.

Hence, the Equations of required lines are $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$.

OR

The lines are $L_1 : \frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $L_2 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Let $P(\lambda-2, 2\lambda+3, 4\lambda-1)$ be any random point on the line L_1 and $Q(2\mu+1, 3\mu+2, 4\mu+3)$ be any random point on the line L_2. Also the given point is $A(1, 1, 1)$.

For some definite values of λ and μ, the required line passes through A, P and Q.

The direction ratios of AP are $\lambda-3,\ 2\lambda+2,\ 4\lambda-2$.

The direction ratios of AQ are $2\mu,\ 3\mu+1,\ 4\mu+2$.

$$\therefore \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}$$

$$\Rightarrow \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{2\lambda-1}{2\mu+1}=k\ \ (\textit{say})$$

$$\Rightarrow \lambda-3=2\mu k,\ 2\lambda+2=3\mu k+k,\ 2\lambda-1=2\mu k+k$$

$$\Rightarrow \mu k=\frac{\lambda-3}{2},\ 2\lambda+2=3\left(\frac{\lambda-3}{2}\right)+k,\ 2\lambda+1=2\left(\frac{\lambda-3}{2}\right)+k$$

$$\Rightarrow \mu k=\frac{\lambda-3}{2},\ k=\frac{\lambda+13}{2},\ k=\lambda+2$$

$$\therefore \frac{\lambda+13}{2}=\lambda+2 \Rightarrow \lambda=9$$

Also, $k=\lambda+2=11$

Hence, the direction ratios of line AP are $6,\ 20,\ 34$ i.e., $3,\ 10,\ 17$.

Therefore, the Cartesian equation of required line is $\dfrac{x-1}{3}=\dfrac{y-1}{10}=\dfrac{z-1}{17}$.

34. Let $I=\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ ...(1)

Using $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$, we get $I=\displaystyle\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$

$$\Rightarrow \quad I=\int_0^{\pi}\frac{(\pi-x)(-\tan x)}{-\sec x-\tan x}dx$$

$$\Rightarrow \quad I=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx \quad ...(2)$$

Adding (1) and (2), we get $2I=\displaystyle\int_0^{\pi}\frac{(x+\pi-x)\tan x}{\sec x+\tan x}dx$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\sin x}dx$$

$$\Rightarrow I=\frac{\pi}{2}\times 2\int_0^{\pi/2}\frac{\sin x}{1+\sin x}dx$$

$\left[\text{Let } f(x)=\dfrac{\sin x}{1+\sin x} \Rightarrow f(\pi-x)=\dfrac{\sin(\pi-x)}{1+\sin(\pi-x)}=\dfrac{\sin x}{1+\sin x}=f(x)\right.$

$$\Rightarrow I=\pi\int_0^{\pi/2}\frac{(1+\sin x)-1}{1+\sin x}dx=\pi\int_0^{\pi/2}\left(1-\frac{1}{1+\sin x}\right)dx=\pi\int_0^{\pi/2}\left(1-\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}\right)dx$$

$$\Rightarrow I=\pi\int_0^{\pi/2}\left(1-\frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx=\pi\int_0^{\pi/2}\left(1-\frac{1}{2}\times\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$$

$$\Rightarrow I = \pi\left[x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I = \pi\left[\left\{\frac{\pi}{2} + \tan 0\right\} - \left\{0 + \tan\frac{\pi}{4}\right\}\right] = \pi\left[\frac{\pi}{2} - 1\right]$$

$$\therefore I = \frac{\pi}{2}[\pi - 2].$$

OR

Let $I = \int_0^{\pi} \log(1 + \cos x)\,dx$ ...(1)

$$\Rightarrow I = \int_0^{\pi} \log[1 + \cos(\pi - x)]\,dx$$

$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x)dx$ ...(2)

Adding (1) and (2), we get $2I = \int_0^{\pi} \log[(1 + \cos x)(1 - \cos x)]\,dx$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi} \log(1 - \cos^2 x)dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi} \log\sin^2 x\,dx = \int_0^{\pi} \log\sin x\,dx$$

Let $f(x) = \log\sin x$. Since $f(\pi - x) = \log[\sin(\pi - x)] = \log\sin x = f(x)$.

$\therefore\ I = 2\int_0^{\pi/2} \log\sin x\,dx$ ...(3) $\left[\text{Using } \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \text{ if } f(2a - x) = f(x)\right.$

$$\Rightarrow I = 2\int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx$$

$\Rightarrow I = 2\int_0^{\pi/2} \log\cos x\,dx$ ...(4)

Adding (3) and (4), we get $2I = 2\int_0^{\pi/2} \log\sin x\cos x\,dx$

$$\Rightarrow I = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right)dx$$

$$\Rightarrow I = \int_0^{\pi/2} \log\sin 2x\,dx - \log 2\int_0^{\pi/2} 1\,dx$$

$\Rightarrow I = I_1 - \log 2[x]_0^{\pi/2} = I_1 - \frac{\pi}{2}\log 2$ ...(5)

Now $I_1 = \int_0^{\pi/2} \log\sin 2x\,dx$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$. Also when $x = 0 \Rightarrow t = 0$ and, when $x = \frac{\pi}{2} \Rightarrow t = \pi$

So, $I_1 = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt = \frac{1}{2}\int_0^{\pi} \log\sin x\,dx$ (Replacing t by x)

$\Rightarrow I_1 = \frac{1}{2}\times 2\int_0^{\pi/2} \log\sin x\,dx$

$\Rightarrow I_1 = \frac{1}{2} I$ [By (3)

From (5), $I = \frac{1}{2}I - \frac{\pi}{2}\log 2$

$\Rightarrow \frac{1}{2}I = -\frac{\pi}{2}\log 2$

$\therefore I = -\pi\log 2$.

35. One-one: Let $x_1, x_2 \in \mathbb{R}_+$ such that $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)\{9(x_1 + x_2) + 6\} = 0$

Either $x_1 - x_2 = 0$ or $9x_1 + 9x_2 + 6 = 0$ which is not possible.

That is, $x_1 = x_2$.

$\therefore f$ is one-one.

Onto: Let $y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - (5 + y) = 0$

$\Rightarrow x = \frac{-6 \pm \sqrt{36 + 4(9)(5+y)}}{2(9)} = \frac{-6 \pm 6\sqrt{1+5+y}}{18}$

$\Rightarrow x = \frac{-1 \pm \sqrt{y+6}}{3}$

Now $x \in \mathbb{R}_+$ i.e., $x \geq 0$ and so, $x = \frac{-1 - \sqrt{y+6}}{3}$ is rejected.

$\therefore x = \frac{-1 + \sqrt{y+6}}{3}$

Now as $x \geq 0$ so, $\frac{-1 + \sqrt{y+6}}{3} \geq 0$

$\Rightarrow \sqrt{y+6} \geq 1$

$\Rightarrow y + 6 \geq 1$

$\Rightarrow y \geq -5$

$\therefore R_f$ (range of f) $= \{y : y \in [-5, \infty)\}$ = Codomain of f.

$\therefore f$ is onto.

Hence, f is one-one and onto function.

SECTION E

36. We have $V(t) = t^3 - 3t^2 + 3t - 100$

(i) No, the above function cannot be used to estimate the number of vehicles in the year 2020, because for 2020 we have $t = 5$ and $V(5) = 5^3 - 3(5^2) + 3(5) - 100 = 125 - 75 + 15 - 100 = -35$, which is not possible.

(ii) $V(25)=(25)^3-3(25)^2+3(25)-100=13725$.

Therefore, the estimated number of vehicles in the year 2040 are 13725.

(iii) $V'(t)=3t^2-6t+3=3(t^2-2t+1)=3(t-1)^2\geq 0$.

Hence, $V(t)$ is always increasing function.

OR

(iii) $V'(t)=3t^2-6t+3$ and $V''(t)=6t-6$

For $V'(t)=0,\ 3t^2-6t+3=0 \quad \Rightarrow 3(t-1)^2=0 \quad \therefore t=1$

Also, $V''(1)=6\times 1-6=0$.

37. **(i)** Required probability $=\dfrac{^{12}C_2}{^{51}C_2}=\dfrac{22}{425}$.

(ii) Required probability $=\dfrac{^{13}C_2}{^{51}C_2}=\dfrac{26}{425}$.

(iii) $\sum_{i=1}^{4} P(A|E_i)=P(A|E_1)+P(A|E_2)+P(A|E_3)+P(A|E_4)$

$$\Rightarrow \quad =\frac{^{12}C_2}{^{51}C_2}+\frac{^{13}C_2}{^{51}C_2}+\frac{^{13}C_2}{^{51}C_2}+\frac{^{13}C_2}{^{51}C_2}$$

$$\Rightarrow \quad =\frac{22}{425}+3\times\frac{26}{425}=\frac{100}{425}=\frac{4}{17}.$$

OR

(iii) P(drawn card is a jack of red colour) $=\dfrac{^{2}C_1}{^{52}C_1}=\dfrac{2}{52}=\dfrac{1}{26}$.

Also P(both drawn cards are king) $=\dfrac{^{4}C_2}{^{52}C_2}=\dfrac{1}{221}$.

38. **(i)** Let length, breadth and height of the tank are x, x and y respectively.

According to the question, $x^2y=500 \quad \Rightarrow y=\dfrac{500}{x^2}$

Surface Area $=S=x^2+4xy$

$$\Rightarrow S=x^2+4x\left(\frac{500}{x^2}\right)=x^2+\frac{2000}{x}$$

Now $\dfrac{dS}{dx}=2x-\dfrac{2000}{x^2}$ and $\dfrac{d^2S}{dx^2}=2+\dfrac{4000}{x^3}$

For maxima or minima, $\dfrac{dS}{dx}=0$

$$\Rightarrow 2x-\frac{2000}{x^2}=0 \quad \therefore x=10 \text{ m}$$

Note that, $\left(\dfrac{d^2S}{dx^2}\right)_{\text{at } x=10}=2+\dfrac{4000}{(10)^3}>0$

$\therefore$ Surface Area is minimum when $x=10$ m.

$\therefore$ Minimum Surface Area $=100+\dfrac{2000}{10}=300\ m^2$.

(ii) If $x = 10$ m then, $y = \dfrac{500}{10^2} = 5$ m.

Volume of the tank $= x^2 y = (10)^2 (5) = 500\ m^3$

New volume $= (2x)^2 y = 4x^2 y = 4(10)^2 (5) = 2000\ m^3$

$\therefore$ Increase in Volume of the tank $= 2000 - 500 = 1500\ m^3$

Therefore, the percentage increase in volume of the tank $= \left(\dfrac{2000 - 500}{500} \times 100 \right)\% = 300\%$.

📖 Detailed Solutions (PTS-22)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. Put $x=\cos\theta \Rightarrow \theta=\cos^{-1}x$.

Also $0<x<\dfrac{1}{\sqrt{2}}$ implies, $0<\cos\theta<\dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{\pi}{4}<\theta<\dfrac{\pi}{2} \Rightarrow \dfrac{\pi}{2}<2\theta<\pi$.

$\therefore \sec^{-1}\left(\dfrac{1}{2x^2-1}\right)=\sec^{-1}\left(\dfrac{1}{2\cos^2\theta-1}\right)=\sec^{-1}\left(\dfrac{1}{\cos 2\theta}\right)=\sec^{-1}\sec 2\theta=2\theta$

Therefore, $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)=2\cos^{-1}x$.

22. Revenue function, $R(x)=3x^2+36x+5$

According to the question, $MR=\dfrac{d}{dx}\left[R(x)\right]$

$\Rightarrow MR=6x+36$

When $x=5$, $MR=6\times5+36=66$.

Hence, the required marginal revenue is ₹66.

23. As $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular so, $\vec{c}.\vec{d}=0$

That is, $(\hat{a}+2\hat{b}).(5\hat{a}-4\hat{b})=0$

$\Rightarrow 5a^2-4\hat{a}.\hat{b}+10\hat{b}.\hat{a}-8b^2=0$ $\quad(\because |\hat{a}|=a=1$ and, $|\hat{b}|=b=1$

$\Rightarrow 6ab\cos\theta=8\times1-5\times1$, where θ is the angle between $\hat{a}$ and $\hat{b}$

$\Rightarrow 6\times1\times1\times\cos\theta=3$

$\Rightarrow \cos\theta=\dfrac{3}{6}=\dfrac{1}{2}$

$\therefore \theta=\dfrac{\pi}{3}$ or, 60°.

OR

The direction ratios of the line $\dfrac{x+3}{3}=\dfrac{y-4}{-5}=\dfrac{z+8}{6}$ are 3, –5, 6.

So, Cartesian equation of required line : $\dfrac{x-(-2)}{3}=\dfrac{y-4}{-5}=\dfrac{z-(-5)}{6}$ i.e., $\dfrac{x+2}{3}=\dfrac{y-4}{-5}=\dfrac{z+5}{6}$.

Also, the vector equation of line : $\vec{r}=-2\hat{i}+4\hat{j}-5\hat{k}+\lambda(3\hat{i}-5\hat{j}+6\hat{k})$.

24. $2^x+2^y=2^{x+y}$

Divide both sides by 2^y, we get $2^{x-y}+1=2^x$ $\quad\ldots$(i)

On differentiating w.r.t. x, we get $2^{x-y}\times\log 2\times\left(1-\frac{dy}{dx}\right)+0=2^x\times\log 2$

$\Rightarrow 2^{x-y}\times\left(1-\frac{dy}{dx}\right)=2^x \qquad \Rightarrow\left(1-\frac{dy}{dx}\right)=\frac{2^x}{2^{x-y}}$

$\Rightarrow\frac{dy}{dx}=1-\frac{2^x}{2^{x-y}}$

$\Rightarrow\frac{dy}{dx}=\frac{2^{x-y}-2^x}{2^{x-y}}=\frac{-1}{2^{x-y}}$ [By (i)]

$\therefore\frac{dy}{dx}=-2^{y-x}$.

OR

As the function $f(x)$ is continuous at $x=3$ so, $\lim\limits_{x\to3}f(x)=f(3)$ i.e., $\lim\limits_{x\to3^+}f(x)=\lim\limits_{x\to3^-}f(x)=f(3)$.

Consider $\lim\limits_{x\to3^+}f(x)=f(3)$ we get, $\lim\limits_{x\to3^+}(bx+3)=a(3)+1$

$\Rightarrow b(3)+3=3a+1$

$\Rightarrow 3a-3b=2$

So, $a-b=\frac{2}{3}$.

25. Consider $\left|\vec{a}+\vec{b}\right|^2=\left\{\left|\vec{a}\right|+\left|\vec{b}\right|\right\}^2$

$\Rightarrow(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=\left\{\left|\vec{a}\right|+\left|\vec{b}\right|\right\}^2$

$\Rightarrow\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\vec{b}.\vec{a}+\vec{a}.\vec{b}=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|$

$\Rightarrow 2\vec{a}.\vec{b}=2\left|\vec{a}\right|\left|\vec{b}\right|$

$\Rightarrow\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\left|\vec{a}\right|\left|\vec{b}\right|$, where θ is the angle between $\vec{a}$ and $\vec{b}$

So, $\cos\theta=1$

$\Rightarrow\theta=0^\circ$.

This implies, $\vec{a}$ and $\vec{b}$ are parallel. So, their d.r.'s will be proportional.

Therefore, $\frac{1}{1}=\frac{\lambda}{1}=\frac{1}{1}$

$\Rightarrow\lambda=1$.

SECTION C

26. Let $I=\int\sec^3 x\,dx$

$\Rightarrow I=\int\sec x\cdot\sec^2 x\,dx$

$\Rightarrow I=\int\sqrt{1+\tan^2 x}\cdot\sec^2 x\,dx$

Put $\tan x=t \Rightarrow \sec^2 x\,dx=dt$

So, $I=\int\sqrt{1+t^2}\,dt$

$\Rightarrow I=\frac{t}{2}\sqrt{1+t^2}+\frac{1}{2}\log\left|t+\sqrt{1+t^2}\right|+C$

$\Rightarrow I=\frac{\sec x\cdot\tan x}{2}+\frac{1}{2}\log\left|\tan x+\sec x\right|+C$.

27. Total numbers which are divisible by 6 from 1 to 100 are 16, nos. which are divisible by 8 are 12 and the nos. which are divisible by 24 are 4.

$\therefore$ P(number is divisible by 6 but not by 24) $= \dfrac{16}{100} - \dfrac{4}{100} = \dfrac{12}{100}$

and P(number is divisible by 8 but not by 24) $= \dfrac{12}{100} - \dfrac{4}{100} = \dfrac{8}{100}$.

Therefore, required probability $= \dfrac{12}{100} + \dfrac{8}{100} = \dfrac{20}{100} = \dfrac{1}{5}$.

OR

Let E_1 and E_2 be the events that the first bag and second bag is selected, respectively.
Let A : the two balls drawn at random, are both red.

Clearly, $P(E_1) = P(E_2) = \dfrac{1}{2}$, $P(A|E_1) = \dfrac{{}^5C_2}{{}^9C_2} = \dfrac{10}{36}$, $P(A|E_2) = \dfrac{{}^3C_2}{{}^9C_2} = \dfrac{3}{36}$

Now $P(E_2|A) = \dfrac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$

$$\therefore P(E_2|A) = \frac{\frac{1}{2} \times \frac{3}{36}}{\frac{1}{2} \times \frac{10}{36} + \frac{1}{2} \times \frac{3}{36}} = \frac{3}{13}.$$

28. Let $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$

Dividing Nr and Dr both by $\cos^2 x$, we get : $I = \displaystyle\int_0^{\pi/2} \frac{1}{1 + 4\tan^2 x} dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt \Rightarrow dx = \dfrac{dt}{1 + \tan^2 x} = \dfrac{dt}{1 + t^2}$.

Also, when $x = 0 \Rightarrow t = 0$ and, when $x \to \dfrac{\pi}{2} \Rightarrow t \to \infty$.

$\therefore \quad I = \displaystyle\int_0^{\infty} \frac{dt}{(1 + 4t^2)(1 + t^2)}$ ...(i)

Consider $\dfrac{1}{(1 + 4t^2)(1 + t^2)} = \dfrac{1}{(1 + 4y)(1 + y)} = \dfrac{A}{1 + 4y} + \dfrac{B}{1 + y}$, where $y = t^2$

$\Rightarrow 1 = A(1 + y) + B(1 + 4y)$

On equating the coefficients of y and constant terms on both sides, we get : $A = \dfrac{4}{3}$, $B = -\dfrac{1}{3}$

Substituting these values in (i), we have : $I = \dfrac{4}{3}\displaystyle\int_0^{\infty} \frac{dt}{(1 + 4t^2)} - \frac{1}{3}\int_0^{\infty} \frac{dt}{(1 + t^2)}$

$\Rightarrow I = \dfrac{4}{3} \cdot \dfrac{1}{2}[\tan^{-1} 2t]_0^{\infty} - \dfrac{1}{3}[\tan^{-1} t]_0^{\infty}$

$\Rightarrow I = \dfrac{2}{3}\left(\dfrac{\pi}{2} - 0\right) - \dfrac{1}{3}\left(\dfrac{\pi}{2} - 0\right) = \dfrac{\pi}{6}$.

OR

Let $I=\int_0^{\frac{\pi}{2}}\sin 2x\ \tan^{-1}(\sin x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}2\sin x\cos x\ \tan^{-1}(\sin x)\,dx$

Put $\sin x=t\Rightarrow\cos x dx=dt$.

Also, when $x=0\Rightarrow t=0$, when $x=\frac{\pi}{2}\Rightarrow t=1$

$\therefore I=\int_0^1 2t\tan^{-1}t\ dt\ \ldots(i)$

Note that $\int 2t\tan^{-1}t\ dt=\tan^{-1}t\int 2t\,dt-\int\left(\frac{d}{dt}(\tan^{-1}t)\int 2t\,dt\right)dt$

$$=t^2\tan^{-1}t-\int\frac{t^2}{1+t^2}dt$$

$$=t^2\tan^{-1}t-\int\left(1-\frac{1}{1+t^2}\right)dt$$

$$=t^2\tan^{-1}t-t+\tan^{-1}t$$

By (i), $I=\left[t^2\tan^{-1}t-t+\tan^{-1}t\right]_0^1$

$\Rightarrow I=\left[1^2\tan^{-1}1-1+\tan^{-1}1\right]-0$

$\therefore\ I=\left(\frac{\pi}{2}-1\right)$.

29. Note that $\frac{dy}{dx}=\frac{2xy-y^2}{2x^2}$ is homogeneous differential equation.

Put $y=vx\ \Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

So, $v+x\frac{dv}{dx}=\frac{2x^2v-v^2x^2}{2x^2}$

$\Rightarrow x\frac{dv}{dx}=\frac{2v-v^2}{2}-v$

$\Rightarrow x\frac{dv}{dx}=\frac{2v-v^2-2v}{2}\qquad\Rightarrow x\frac{dv}{dx}=-\frac{v^2}{2}$

$\Rightarrow-\int\frac{2\,dv}{v^2}=\int\frac{dx}{x}$

$\Rightarrow\frac{2}{v}=\log|x|+C$

$\Rightarrow\frac{2x}{y}=\log|x|+C\ \ldots(i)$

Comparing (i) with $\frac{ax}{y}=b\log|x|+C$, we get : $a=2,\ b=1$.

OR

$y\,e^y dx=(y^3+2xe^y)\,dy$

$\Rightarrow y \cdot e^y \frac{dx}{dy} = y^3 + 2xe^y$

$\therefore \frac{dx}{dy} - \frac{2}{y}x = y^2 \cdot e^{-y}$

On comparing $\frac{dx}{dy} - \frac{2}{y}x = y^2 \cdot e^{-y}$ with $\frac{dx}{dy} + Px = Q$, we get $P = -\frac{2}{y}$, $Q = y^2 \cdot e^{-y}$

Now, I.F. (Integrating factor) $= e^{-2\int \frac{1}{y} dy} = e^{-2\log y} = e^{\log y^{-2}} = \frac{1}{y^2}$

$\therefore$ Solution is : $x \cdot \frac{1}{y^2} = \int y^2 \cdot e^{-y} \cdot \frac{1}{y^2} dy + C$

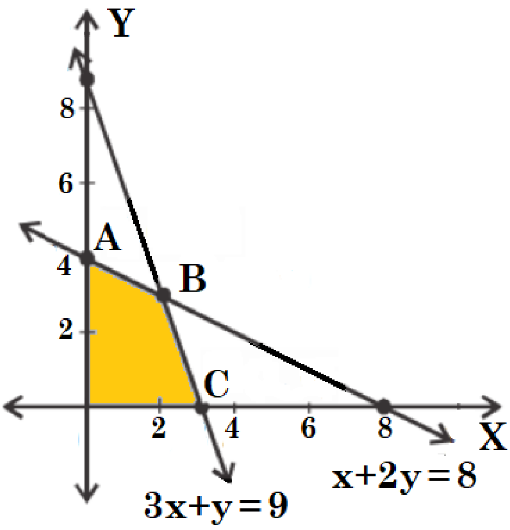
$\Rightarrow \frac{x}{y^2} = \int e^{-y} \, dy + C$

$\Rightarrow \frac{x}{y^2} = -e^{-y} + C$

$\therefore x = -y^2 e^{-y} + C y^2$.

30. Consider the graph shown here.

Corner Points	Value of Z
A(0, 4)	600
B(2, 3)	690 $\leftarrow$ max.
C(3, 0)	360
O(0, 0)	0



So, maximum value of Z is 690.
The coordinates of the point at which Z_{max} occurs is (2, 3).

31. Let $I = \int \frac{x}{(x-1)^2(x+2)} dx$

Consider $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

On comparing the coefficients of like terms both sides, we get

$A = \frac{2}{9}, B = \frac{1}{3}, C = -\frac{2}{9}$

$\therefore I = \int \left(\frac{2}{9} \times \frac{1}{x-1} + \frac{1}{3} \times \frac{1}{(x-1)^2} - \frac{2}{9} \times \frac{1}{x+2} \right) dx$

$\Rightarrow I = \frac{2}{9} \times \log|x-1| - \frac{1}{3} \times \frac{1}{(x-1)} - \frac{2}{9} \times \log|x+2| + C$

Therefore, $I = \frac{2}{9} \times \log\left|\frac{x-1}{x+2}\right| - \frac{1}{3} \times \frac{1}{(x-1)} + C$.

SECTION D

32. Equation of the line joining A(1, 3) and B(0, 0) : $\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$

Expanding along R_3, we get : $3x - y = 0$.

As $ar(ABD) =$ Magnitude of $\frac{1}{2}\begin{vmatrix} m & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 3$

That is, $\frac{1}{2}\begin{vmatrix} m & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 3$

Expanding along R_3, we get : $3m = \pm 6$

So, $m = \pm 2$.

33. We have $x = y^2$, $x = 4$ and $x = k$.

As per the question, we have

$ar(OAEO) = ar(ABCDEFA)$

$\Rightarrow 2ar(OAFO) = 2\,ar(ABCFA)$

$\Rightarrow \int_0^k \sqrt{x}dx = \int_k^4 \sqrt{x}dx$

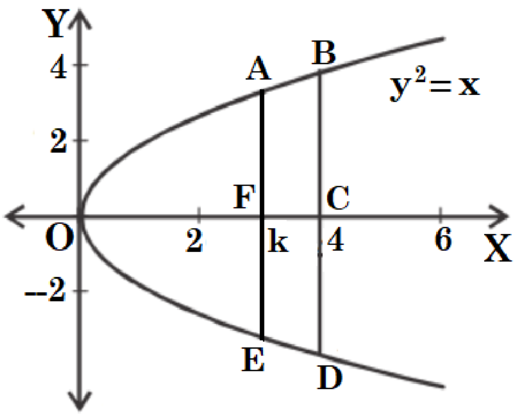
$\Rightarrow \frac{2}{3}\left[x^{3/2}\right]_0^k = \frac{2}{3}\left[x^{3/2}\right]_k^4$

$\Rightarrow k^{3/2} = 4^{3/2} - k^{3/2}$

$\Rightarrow 2 \times k^{3/2} = 8$

$\Rightarrow k^{3/2} = 4$

$\therefore k = 4^{2/3}$.



34. Since $(x, x) \in R$ as a word will have 'all letters in common' with itself.

So, R is reflexive.

Also if x and y are words with 'at least one letter in common', then we will surely have $(x, y) \in R$ as well as $(y, x) \in R$. So, R is symmetric.

Let x be a word 'Cat', y be a word 'Apple' and z be a word 'Pen'.

Note that $(x, y) \in R$ and $(y, z) \in R$ as 'Cat' and 'Apple' have letter "a" in common and 'Apple' and 'Pen' have letters "p" and "e" in common.

But $(x, z) \notin R$ as 'Cat' and 'Pen' doesn't have any letter in common.

So, R isn't transitive.

OR

Let $A = \mathbb{R} - \{-1\}$, $B = \mathbb{R} - \{1\}$.

So, we have $f : A \to B$, $f(x) = \frac{x}{x+1}$.

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$.

That is, $\frac{x_1}{x_1 + 1} = \frac{x_2}{x_2 + 1}$

$\Rightarrow x_1x_2 + x_1 = x_1x_2 + x_2$
$\Rightarrow x_1 = x_2$
$\therefore f(x)$ is one-one.
Let $y = f(x),\ y \in B$.
That is, $y = \dfrac{x}{x+1}$
$\Rightarrow xy + y = x$
$\Rightarrow y = x - xy$
$\Rightarrow x = \dfrac{y}{1-y},\ y \neq 1$
That implies, $y \in B = \mathbb{R} - \{1\}\ \forall\ x \in A = \mathbb{R} - \{-1\}$.
Therefore, Range $=$ Codomain .
$\therefore f(x)$ is onto.

35. $f(x) = \tan x - 4x$

So, $f'(x) = \sec^2 x - 4$

(a) For $f(x)$ to be strictly increasing

$f'(x) > 0$
$\Rightarrow \sec^2 x - 4 > 0$
$\Rightarrow \sec^2 x > 4$
$\Rightarrow \dfrac{1}{\cos^2 x} > 4$
$\Rightarrow \cos^2 x < \dfrac{1}{4}$
$\Rightarrow \cos^2 x < \left(\dfrac{1}{2}\right)^2$
$\Rightarrow -\dfrac{1}{2} < \cos x < \dfrac{1}{2}$ $\left\{\because x \in \left(0, \dfrac{\pi}{2}\right)\right.$
$\Rightarrow \dfrac{\pi}{3} < x < \dfrac{\pi}{2}$

Hence, $f(x)$ is strictly increasing in $x \in \left(\dfrac{\pi}{3}, \dfrac{\pi}{2}\right)$.

(b) For $f(x)$ to be strictly decreasing

$f'(x) < 0$
$\Rightarrow \sec^2 x - 4 < 0$
$\Rightarrow \sec^2 x < 4$
$\Rightarrow \dfrac{1}{\cos^2 x} < 4$
$\Rightarrow \cos^2 x > \dfrac{1}{4}$
$\Rightarrow \cos^2 x > \left(\dfrac{1}{2}\right)^2$

$\Rightarrow \cos x > \frac{1}{2}$ $\left\{\because x \in \left(0, \frac{\pi}{2}\right)\right.$

$\Rightarrow 0 < x < \frac{\pi}{3}$

Hence, $f(x)$ is strictly decreasing in $x \in \left(0, \frac{\pi}{3}\right)$.

OR

Let l and b represent the length of rectangle and its width respectively.
So, $2(l + b) = 36$
$\Rightarrow l + b = 18 \ldots$(i)
If rectangle is revolved about its length l to form a cylinder then, radius of the base of cylinder will be $r = b$ and height $h = l$.
Now volume of cylinder, $V = \pi r^2 h$
$\Rightarrow V = \pi b^2(18 - b) = \pi(18b^2 - b^3)$

$\therefore \frac{dV}{db} = \pi(36b - 3b^2) = 3\pi(12b - b^2)$ and, $\frac{d^2V}{db^2} = 3\pi(12 - 2b) = 6\pi(6 - b)$

For critical points, $\frac{dV}{db} = 0 \quad \Rightarrow 3\pi(12b - b^2) = 0$
$\Rightarrow b(12 - b) = 0$
$\because b \neq 0 \quad \therefore b = 12$

Note that $\left.\frac{d^2V}{db^2}\right]_{\text{at } b=12} = -36\pi < 0$

So, V is maximum when $b = 12$ cm.
Also by (i), $l + b = 18$ so, $l = 6$
Hence, $l = 6$ cm, $b = 12$ cm are the dimensions of rectangle.

SECTION E

36. **(i)** The direction ratios of the line $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ are 1, 2, –1.

Also, the direction ratios of the line $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ are 2, 1, 1.

(ii) Comparing with $\vec{r} = \vec{a} + u\vec{b}$, we get $\vec{a} = 3\hat{i} + 3\hat{j}$.
So, the point is (3, 3, 0).

(iii) For the given lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$, we have

$\vec{a}_1 = \vec{0}, \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{a}_2 = 3\hat{i} + 3\hat{j}, \vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

Now $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

As $S.D. = \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$\Rightarrow \text{S.D.} = \dfrac{\left|(3\hat{i}+3\hat{j}).(3\hat{i}-3\hat{j}-3\hat{k})\right|}{\left|3\hat{i}-3\hat{j}-3\hat{k}\right|}$

$\Rightarrow \text{S.D.} = \dfrac{|9-9|}{\sqrt{9+9+9}}$

$\therefore$ S.D. = 0 units.

That is, the lines will intersect each other.

OR

(iii) Re-writing the given lines in Cartesian form, $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{-1}=\lambda, \dfrac{x-3}{2}=\dfrac{y-3}{1}=\dfrac{z}{1}=\mu$

The coordinates of random point on these lines are $P(\lambda, 2\lambda, -\lambda)$ and $Q(2\mu+3, \mu+3, \mu)$.

If the lines intersect then, the points P and Q must coincide.

That is, $\lambda = 2\mu+3$...(i), $2\lambda = \mu+3$...(ii), $-\lambda = \mu$...(iii)

On solving (i) and (iii), we get : $-\mu = 2\mu+3 \Rightarrow \mu=-1, \lambda=1$.

Note that, LHS of (ii): $2\lambda = 2\times1=2$, RHS of (ii): $\mu+3=-1+3=2$. So, LHS = RHS.

Therefore, the lines will intersect each other.

Also, the required point of intersection : (1, 2, –1).

So, the motorcycles may collide at (1, 2, –1).

37. **(i)** Let E : the chosen item is defective.

Also let E_1, E_2, E_3 be the events that the item was produced by operators A, B, C respectively.

$\therefore$ $P(E|E_1)=1\%$.

(ii) Refer to (i).

Here $x=P(E|E_2)=5\%$, $y=P(E|E_3)=7\%$

$\therefore$ $x+y=5\%+7\%$

$\Rightarrow x+y=12\%$.

(iii) Refer to (i).

Here $P(E_1)=50\%$, $P(E_2)=30\%$, $P(E_3)=20\%$, $P(E|E_1)=1\%$, $P(E|E_2)=5\%$, $P(E|E_3)=7\%$.

So, $P(E)=P(E|E_1)P(E_1)+P(E|E_2)P(E_2)+P(E|E_3)P(E_3)$

$\Rightarrow P(E)=\dfrac{1}{100}\times\dfrac{50}{100}+\dfrac{5}{100}\times\dfrac{30}{100}+\dfrac{7}{100}\times\dfrac{20}{100}$

$\Rightarrow P(E)=\dfrac{34}{1000}=3.4\%$.

OR

(iii) Refer to (i) and (iii).

Using Bayes' theorem, $P(E_1|E)=\dfrac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)+P(E|E_3)P(E_3)}$

$\Rightarrow P(E_1|E)=\dfrac{\frac{1}{100}\times\frac{50}{100}}{\frac{1}{100}\times\frac{50}{100}+\frac{5}{100}\times\frac{30}{100}+\frac{7}{100}\times\frac{20}{100}}=\dfrac{5}{5+15+14}=\dfrac{5}{34}$.

38. **(i)** Since capacity of the tank is, $250\text{ m}^3 = x\times x\times h \Rightarrow h=\dfrac{250}{x^2}$

Total cost, $C=5000\times x^2+40000\times h^2$

$\Rightarrow C = 5000 \times x^2 + 40000 \times \left(\frac{250}{x^2}\right)^2$

$\Rightarrow C = 5000 \times x^2 + \frac{2500000000}{x^4}$

Now $\frac{dC}{dx} = 5000 \times 2x - \frac{4 \times 2500000000}{x^5} = 10000x - \frac{10000000000}{x^5}$

Also $\frac{d^2C}{dx^2} = 10000 + \frac{50000000000}{x^6}$

For $\frac{dC}{dx} = 0,\ 10000x - \frac{10000000000}{x^5} = 0$

$\Rightarrow 10000\left[x - \frac{1000000}{x^5}\right] = 0$

$\Rightarrow \left[\frac{x^6 - 1000000}{x^5}\right] = 0$

$\Rightarrow x^6 - 1000000 = 0 \quad \Rightarrow x^6 = 10^6$

$\therefore x = 10$ m.

Note that, $\left(\frac{d^2C}{dx^2}\right)_{\text{at } x=10} = 10000 + \frac{50000000000}{10^6} = 10000 + 50000 = 60000 > 0$ so, C is minimum when $x = 10$ m.

(ii) Note that, $\frac{dC}{dx} = 10000x - \frac{10000000000}{x^5} = 10000\left[\frac{x^6 - 10^6}{x^5}\right]$

Clearly when $x > 0, \frac{dC}{dx} < 0$.

That means, $C(x)$ is not an increasing function, when $x > 0$.

Detailed Solutions (PTS-23)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. $\cos^{-1}\cos 320^\circ - \sin^{-1}\sin 320^\circ = \cos^{-1}\cos(360^\circ - 40^\circ) - \sin^{-1}\sin(360^\circ - 40^\circ)$

$\Rightarrow \quad = \cos^{-1}\cos 40^\circ - \sin^{-1}[-\sin 40^\circ]$

$\Rightarrow \quad = 40^\circ + \sin^{-1}\sin 40^\circ = 80^\circ$.

OR

Here $R : Z \to Z$, aRb if and only if $a^2 - 7ab + 6b^2 = 0$.

Reflexivity : As $a^2 - 7a.a + 6a^2 = 7a^2 - 7a^2 = 0 \ \forall \ a \in Z$.

So, $(a, a) \in R \ \forall \ a \in Z$. Hence, R is reflexive.

Symmetry : Let $(6,1) \in R$. Clearly, $6^2 - 7.6.1 + 6.1^2 = 42 - 42 = 0$

But $(1,6) \notin R$ as, $1^2 - 7.1.6 + 6.6^2 = 1 - 42 + 126 = 175 \neq 0$.

So, R is not symmetric.

22. $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$

$$\Rightarrow \frac{dy}{d\theta} = \frac{(2+\cos\theta)4\cos\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{8\cos\theta + 4(\sin^2\theta + \cos^2\theta) - 4 - \cos^2\theta - 4\cos\theta}{(2+\cos\theta)^2}$$

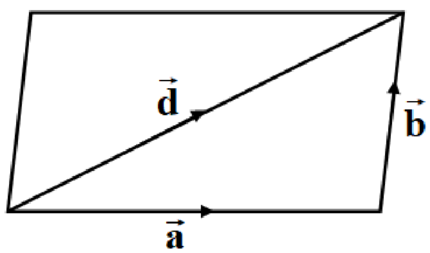
$$\Rightarrow \frac{dy}{d\theta} = \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$$

As $\dfrac{dy}{d\theta} = \left[\dfrac{4-\cos\theta}{(2+\cos\theta)^2}\right]\cos\theta \geq 0$ for all $\theta \in \left[0, \dfrac{\pi}{2}\right]$ so, y is an increasing function of θ on $\left[0, \dfrac{\pi}{2}\right]$.

23. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ be the side and $\vec{d} = 4\hat{i} + 5\hat{k}$ be the diagonal of parallelogram.

$\because \ \vec{a} + \vec{b} = \vec{d}$, where $\vec{b}$ is the other side of parallelogram.

$\therefore \vec{b} = \vec{d} - \vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$.



Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

$\Rightarrow \vec{a} \times \vec{b} = -5\hat{i} - \hat{j} + 4\hat{k}$

So, the area of parallelogram $=|\vec{a}\times\vec{b}|=\sqrt{25+1+16}$

$=\sqrt{42}$ Sq. units .

OR

Cartesian equation of the line joining two points (1, 2, 3) and (–3, 4, 3) is $\frac{x-1}{-3-1}=\frac{y-2}{4-2}=\frac{z-3}{3-3}$

$\Rightarrow \frac{x-1}{-4}=\frac{y-2}{2}=\frac{z-3}{0}$ i.e., $\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z-3}{0}$ is the required Cartesian equation of line.

Now the d.r.'s of line obtained above are –2, 1, 0.

Also the d.r.'s of z-axis are 0, 0, 1.

Since $(-2)(0)+(1)(0)+(0)(1)=0$ so, clearly line obtained above is perpendicular to the z-axis.

(By using $a_1a_2+b_1b_2+c_1c_2=0$, condition for perpendicular lines).

24. We have $x=t+\frac{1}{t}$ and, $y=t-\frac{1}{t}$

$\Rightarrow \frac{dx}{dt}=\frac{d}{dt}\left(t+\frac{1}{t}\right)=1-\frac{1}{t^2}$, and $\frac{dy}{dt}=1+\frac{1}{t^2}$

$\because \frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ $\quad\therefore \frac{dy}{dx}=\frac{t^2+1}{t^2}\times\frac{t^2}{t^2-1}=\frac{t^2+1}{t^2-1}$

Therefore, $\left(\frac{dy}{dx}\right)_{\text{at } t=2}=\frac{2^2+1}{2^2-1}=\frac{5}{3}$.

25. As $|\sqrt{2}\,\hat{a}-\hat{b}|=1$

So, $|\sqrt{2}\,\hat{a}-\hat{b}|^2=1^2$ implies, $(\sqrt{2}\,\hat{a}-\hat{b}).(\sqrt{2}\,\hat{a}-\hat{b})=1$

$\Rightarrow 2|\hat{a}|^2+|\hat{b}|^2-2\sqrt{2}\,\hat{a}.\hat{b}=1$

$\Rightarrow 2\times1^2+1^2-2\sqrt{2}|\hat{a}||\hat{b}|\cos\theta=1$, where θ is the required angle between $\hat{a}$ and $\hat{b}$

$\Rightarrow -2\sqrt{2}\times1\times1\times\cos\theta=-2$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$

$\therefore \theta=\frac{\pi}{4}$.

SECTION C

26. Let $I=\int\frac{x^2\,(x^2+1)}{(x^2+2)(x^2+3)}dx=\int\frac{x^4+x^2}{x^4+5x^2+6}dx=\int\left(1-\frac{4x^2+6}{x^4+5x^2+6}\right)dx$

$\Rightarrow I=\int\left(1-\frac{4x^2+6}{(x^2+2)(x^2+3)}\right)dx=\int 1dx-\int\frac{4x^2+6}{(x^2+2)(x^2+3)}dx$ ...(i)

Consider $\frac{4x^2+6}{(x^2+2)(x^2+3)}=\frac{4t+6}{(t+2)(t+3)}=\frac{A}{t+2}+\frac{B}{t+3}$, where $t=x^2$

$\Rightarrow 4t+6=A(t+3)+B(t+2)$

On comparing the like terms on both sides, we get $A=-2, B=6$

By (i), $I=\int 1dx-\int\left[\frac{-2}{(x^2+2)}+\frac{6}{(x^2+3)}\right]dx$

$\Rightarrow I=x-\left[-2\times\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+6\times\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right]+C$

$\therefore I=x+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}-2\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+C$.

27. Let G: the seed germinates, E_1: flowers seeds of type A_1, E_2: flowers seeds of type A_2 and, E_3: flowers seeds of type A_3.

So, $P(E_1)=\frac{4}{10}$, $P(E_2)=\frac{4}{10}$, $P(E_3)=\frac{2}{10}$, $P(G|E_1)=\frac{45}{100}$, $P(G|E_2)=\frac{60}{100}$, $P(G|E_3)=\frac{35}{100}$.

$\therefore P(G)=P(E_1).P(G|E_1)+P(E_2).P(G|E_2)+P(E_3).P(G|E_3)$

$\Rightarrow P(G)=\frac{4}{10}.\frac{45}{100}+\frac{4}{10}.\frac{60}{100}+\frac{2}{10}.\frac{35}{100}$

$\therefore P(G)=\frac{49}{100}=0.49$.

OR

Let E_1: letter has come from the city CHENNAI, E_2: letter has come from the city JAIPUR, E_3: letter has come from the city NAINITAL, E_4: letter has come from the city BEGUSARAI, and E_5: letter has come from the city MUMBAI.

Also let E: two consecutive letters AI are legible on the envelope.

Now $P(E_1)=P(E_2)=P(E_3)=P(E_4)=P(E_5)=\frac{1}{5}$,

$P(E|E_1)=\frac{n(E\cap E_1)}{n(E_1)}=\frac{1}{6}$ (As 6 pairs of consecutive letters are CH, HE, EN, NN, NA, AI;

$P(E|E_2)=\frac{n(E\cap E_2)}{n(E_2)}=\frac{1}{5}$ (As 5 pairs of consecutive letters are JA, AI, IP, PU, UR;

$P(E|E_3)=\frac{n(E\cap E_3)}{n(E_3)}=\frac{1}{7}$ (As 7 pairs of consecutive letters are NA, AI, IN, NI, IT, TA, AL;

$P(E|E_4)=\frac{n(E\cap E_4)}{n(E_4)}=\frac{1}{8}$ (As 8 pairs of consecutive letters are BE, EG, GU, US, SA, AR, RA, AI;

$P(E|E_5)=\frac{n(E\cap E_5)}{n(E_5)}=\frac{1}{5}$ (As 5 pairs of consecutive letters are MU, UM, MB, BA, AI.

$\because P(E)=P(E|E_1)\ P(E_1)+P(E|E_2)\ P(E_2)+P(E|E_3)\ P(E_3)+P(E|E_4)\ P(E_4)+P(E|E_5)\ P(E_5)$

$\therefore P(E)=\frac{1}{5}\left[\frac{1}{6}+\frac{1}{5}+\frac{1}{7}+\frac{1}{8}+\frac{1}{5}\right]=\frac{1}{5}\times\frac{701}{840}$

Hence required probability, $P(E_5|E)=\frac{P(E|E_5)\ P(E_5)}{P(E)}$ (using Bayes' theorem

$\Rightarrow P(E_5|E)=\frac{\frac{1}{5}\times\frac{1}{5}}{\frac{1}{5}\times\frac{701}{840}}=\frac{1}{5}\times\frac{840}{701}=\frac{168}{701}$.

28. Consider $f(x)=\dfrac{\cos 2x}{1+\cos 2x}$

$\Rightarrow f(-x)=\dfrac{\cos 2(-x)}{1+\cos 2(-x)}=\dfrac{\cos 2x}{1+\cos 2x}=f(x)$

That means, $f(x)$ is an even function.

So, by using $\int_{-a}^{a} f(x)\,dx=2\int_0^a f(x)\,dx$, if $f(-x)=f(x)$ we get

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}dx=2\int_0^{\frac{\pi}{4}}\frac{\cos 2x}{1+\cos 2x}dx=2\int_0^{\frac{\pi}{4}}\left(1-\frac{1}{1+\cos 2x}\right)dx$$

$$=2\int_0^{\frac{\pi}{4}}\left(1-\frac{1}{2\cos^2 x}\right)dx=2\int_0^{\frac{\pi}{4}}\left(1-\frac{\sec^2 x}{2}\right)dx$$

$$=2\left[x-\frac{\tan x}{2}\right]_0^{\pi/4}$$

$$=2\left\{\left[\frac{\pi}{4}-\frac{1}{2}\right]-0\right\}=\left(\frac{\pi}{2}-1\right).$$

OR

$\because \dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{1}{x}\right)\right]=\dfrac{1}{1+\left(\dfrac{1}{x}\right)^2}\times\left(-\dfrac{1}{x^2}\right)=-\dfrac{1}{x^2+1}.$

Now let $I=\int_{-1}^{1}\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{1}{x}\right)\right]dx$

$\Rightarrow I=\int_{-1}^{1}\left\{-\dfrac{1}{1+x^2}\right\}dx$

Consider $f(x)=\dfrac{1}{1+x^2}$

$\Rightarrow f(-x)=\dfrac{1}{1+(-x)^2}=\dfrac{1}{1+x^2}=f(x)$

So, $f(x)$ is an even function.

By using $\int_{-a}^{a} f(x)\,dx=\begin{cases}2\int_0^a f(x)\,dx, \text{ if } f(x) \text{ is even function i.e., } f(-x)=f(x)\\ 0, \text{ if } f(x) \text{ is odd function i.e., } f(-x)=-f(x)\end{cases}$

$\therefore I=-2\int_0^1\dfrac{1}{1+x^2}dx$

$\Rightarrow I=-2\left[\tan^{-1}x\right]_0^1$

$\Rightarrow I=-2\left[\tan^{-1}1-\tan^{-1}0\right]$

$\Rightarrow I=-2\left[\tan^{-1}\tan\dfrac{\pi}{4}-0\right]=-\dfrac{\pi}{2}.$

Note that, $\int_{-1}^{1} \frac{d}{dx}\left[\tan^{-1}\left(\frac{1}{x}\right)\right]dx = \left[\tan^{-1}\left(\frac{1}{x}\right)\right]_{-1}^{1} = \frac{\pi}{2}$.

It is not correct as $\frac{1}{x}$ is not defined at $x = 0 \in (-1, 1)$.

29. Let $I = \int (3x+5)\sqrt{5+4x-2x^2}\,dx$

Consider $3x+5 = A\frac{d}{dx}\left[5+4x-2x^2\right] + B$

$\Rightarrow 3x+5 = A[4-4x] + B$

Comparing the like terms, we get $A = -\frac{3}{4}$, $B = 8$.

Therefore, $I = -\frac{3}{4}\int (4-4x)\sqrt{5+4x-2x^2}\,dx + 8\int \sqrt{5+4x-2x^2}\,dx$

Put $5+4x-2x^2 = t \Rightarrow (4-4x)dx = dt$ in the 1st integral.

So, $I = -\frac{3}{4}\int \sqrt{t}\,dt + 8\int \sqrt{-2\left(x^2-2x-\frac{5}{2}\right)}dx$

$\Rightarrow I = -\frac{3}{4}\times\frac{2}{3}(t)^{3/2} + 8\sqrt{2}\int \sqrt{-\left((x-1)^2-\frac{7}{2}\right)}dx$

$\Rightarrow I = -\frac{1}{2}(t)^{3/2} + 8\sqrt{2}\int \sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - (x-1)^2}\,dx$

$\Rightarrow I = -\frac{1}{2}(t)^{3/2} + 8\sqrt{2}\left[\frac{(x-1)}{2}\sqrt{\frac{5}{2}+2x-x^2} + \frac{7}{4}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}\right] + C$

$\therefore I = -\frac{1}{2}(5+4x-2x^2)^{3/2} + 4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2} + 14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}} + C$.

30. $\frac{dy}{dx} - y\tan x = 2\sin x$

On comparing with $\frac{dy}{dx} + P(x)y = Q(x)$, we have $P(x) = -\tan x$, $Q(x) = 2\sin x$

Integration factor $= e^{\int -\tan x\,dx} = e^{\log\cos x} = \cos x$.

So, the solution is given by $y(\cos x) = \int 2\sin x\cos x\,dx + C$

$\Rightarrow y(\cos x) = \int \sin 2x\,dx + C$

$\Rightarrow y(\cos x) = -\frac{1}{2}\cos 2x + C$.

Therefore, the required solution is $y = \frac{k-\cos 2x}{2\cos x}$, where $2C = k$.

OR

$\frac{dy}{dx} = \frac{1}{\log y - \log x} + \frac{y}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log\left(\frac{y}{x}\right)} + \frac{y}{x}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.

So, $v + x\frac{dv}{dx} = \frac{1}{\log v} + v$

$$\Rightarrow \int \log v\, dv = \int \frac{dx}{x}$$

$$\Rightarrow \log v \int 1\, dv - \int \left(\frac{d}{dv}(\log v) \int 1\, dv \right) dv = \int \frac{dx}{x}$$

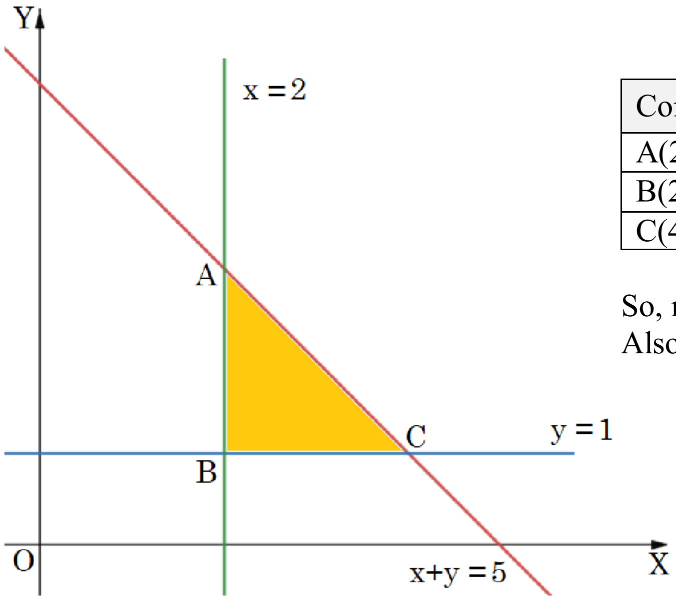
$$\Rightarrow v\log v - \int \left(\frac{1}{v} \times v \right) dv = \int \frac{dx}{x}$$

$$\Rightarrow v\log v - v = \log x + C$$

$$\Rightarrow \frac{y}{x} \times \left(\log\frac{y}{x} - 1 \right) = \log x + C$$

That is, $y(\log y - \log x - 1) = x(\log x + C)$.

31. Consider the graph shown below.



Corner Points	Value of Z
A(2, 3)	57 ← Max.
B(2, 1)	29 ← Min.
C(4, 1)	49

So, maximum value of Z is 57.
Also, the minimum value of Z is 29.

SECTION D

32. Given $f : W \rightarrow W$ given by $f(n) = \begin{cases} n+1, \text{ if } n \text{ is even} \\ n-1, \text{ if } n \text{ is odd} \end{cases}$.

For one-one : Let $n, m \in W$.

If n and m are both even, then $f(n) = f(m) \Rightarrow n+1 = m+1 \Rightarrow n = m$

If n and m are both odd, then $f(n) = f(m) \Rightarrow n-1 = m-1 \Rightarrow n = m$

Thus in both the cases, we've $f(n) = f(m) \Rightarrow n = m$.

If n is odd and m is even then, $f(n) = n-1$ is even and $f(m) = m+1$ is odd.

Therefore, $n \neq m \Rightarrow f(n) \neq f(m)$.

Similarly if n is even and m is odd, then also $n \neq m \Rightarrow f(n) \neq f(m)$.

Hence, f is one-one.

For onto : Let $n \in W$ be an arbitrary element. If n is an odd natural number, then $\exists$ an even natural number $n-1 \in W$ (domain) such that $f(n-1) = n-1+1 = n$.

Also if n is an even natural number, then $\exists$ an odd natural number $n+1 \in W$ (domain) such that $f(n+1) = n+1-1 = n$.

Also note that $f(1) = 0$.

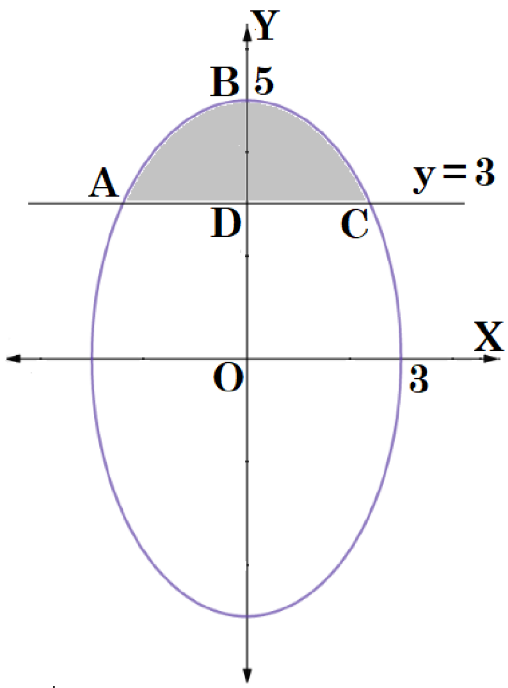
So every element of W (codomain) has its pre-image in the domain W.

Hence, f is onto.

33. We have $\frac{x^2}{9} + \frac{y^2}{25} = 1$ and $y = 3$.

$\Rightarrow \frac{x^2}{9} = 1 - \frac{y^2}{25}$

$\Rightarrow x = \pm\frac{3}{5} \times \sqrt{25 - y^2}$



Note that the given ellipse is symmetrical about both the axes.

Required area $=$ ar(ABCDA)

$\Rightarrow \quad = 2 \times$ ar(BCDB)

$\Rightarrow \quad = 2 \times \int_3^5 \frac{3}{5}\sqrt{25 - y^2}\, dy$

$\Rightarrow \quad = \frac{6}{5}\left[\frac{y}{2}\sqrt{25 - y^2} + \frac{25}{2}\sin^{-1}\frac{y}{5}\right]_3^5$

$\Rightarrow \quad = \frac{6}{5}\left[\left(\frac{5}{2}\sqrt{25 - 5^2} + \frac{25}{2}\sin^{-1}\frac{5}{5}\right) - \left(\frac{3}{2}\sqrt{25 - 3^2} + \frac{25}{2}\sin^{-1}\frac{3}{5}\right)\right]$

$\Rightarrow \quad = \frac{6}{5}\left[\left(0 + \frac{25}{2}\sin^{-1}1\right) - \left(6 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right)\right]$

$\Rightarrow \quad = \left(\frac{15\pi}{2} - \frac{36}{5} - 15\sin^{-1}\frac{3}{5}\right)$ Sq.units.

OR

Given $|x| + |y| = 1$

$\therefore$ When $x \geq 0, y \geq 0$ then $x + y = 1 \ldots$(i),

When $x \leq 0, y \geq 0$ then $-x + y = 1 \ldots$(ii),

When $x \leq 0, y \leq 0$ then $-x - y = 1 \ldots$(iii),

When $x \geq 0, y \leq 0$ then $x - y = 1 \ldots$(iv).

Also the curve is symmetrical about both axes.

$\therefore$ Required area $=$ ar(ABCDA)

$\Rightarrow \quad = 4 \times$ ar(ABOA)

$\Rightarrow \quad = 4\left(\int_0^1 y_1 dx\right)$

$\Rightarrow \quad = 4\int_0^1 (1-x)dx$

$\Rightarrow \quad = \frac{4}{-2}\left[(1-x)^2\right]_0^1$

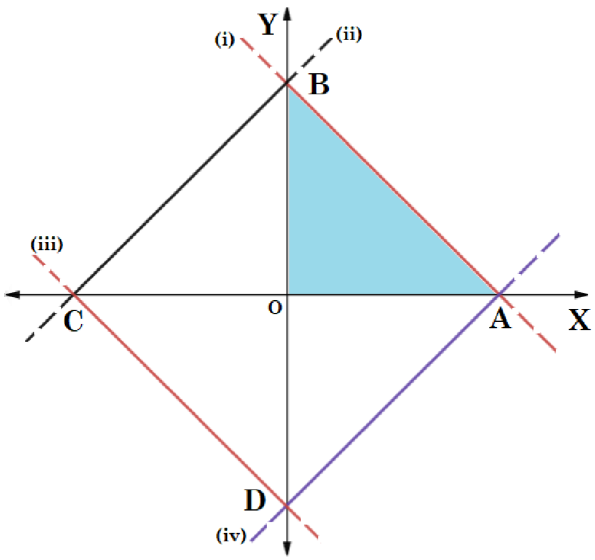
$\Rightarrow \quad = -2\left[(1-1)^2-(1-0)^2\right]$

$\Rightarrow \quad = -2\left[0^2-1^2\right]$

$\Rightarrow \quad = -2[0-1]$

$\Rightarrow \quad = (-2)(-1)$

$\Rightarrow \quad = 2$ Sq.units.



34. We have $L_1 : \frac{x-6}{1} = \frac{2-y}{2} = \frac{z-2}{2} = \lambda$ (say) and $L_2 : \frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2} = \mu$ (say).

Let the line of S.D. meets the given lines L_1 and L_2 at P and Q respectively.

So, the coordinates of any random point on the liens are given as :

$P(6+\lambda, 2-2\lambda, 2+2\lambda)$ and $Q(-4+3\mu, -2\mu, -1-2\mu)$.

The d.r.'s of PQ are $\lambda-3\mu+10,\ -2\lambda+2\mu+2,\ 2\lambda+2\mu+3$.

Since the line of S.D. (line PQ) is perpendicular to both the given lines.

So by using $a_1a_2+b_1b_2+c_1c_2=0$, we have : $3\lambda-\mu+4=0$ and $3\lambda-17\mu+20=0$

On solving these equations, we get : $\lambda=-1,\ \mu=1$.

$\therefore$ Coordinates of the required points are $P(5,4,0)$ and $Q(-1,-2,-3)$.

Hence length of S.D. is $PQ=\sqrt{(-1-5)^2+(-2-4)^2+(-3-0)^2}=\sqrt{36+36+9}=9$ units.

And, the equation of S.D. (line PQ) is : $\frac{x-5}{-1-5}=\frac{y-4}{-2-4}=\frac{z-0}{-3-0}$ i.e., $\frac{x-5}{2}=\frac{y-4}{2}=\frac{z-0}{1}$.

OR

Assume P(–1, 6, 3). Let M be foot of perpendicular on the given line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$.

Any arbitrary point on the line is $M(\lambda,\ 2\lambda+1,\ 3\lambda+2)$.

The direction ratios of PM : $\lambda+1,\ 2\lambda-5,\ 3\lambda-1$.

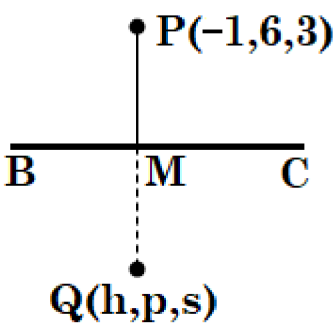
Since PM is perpendicular to the given line.

So using $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp$ lines, we've :

$1(\lambda+1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow 14\lambda-12=0$

$\Rightarrow \lambda=\frac{6}{7}$



$\therefore M\left(\frac{6}{7},\frac{19}{7},\frac{32}{7}\right)$ is the **foot of perpendicular** of the point P in line.

Let image of P in the line be $Q(h, p, s)$.

Since M is the mid-point of PQ, so $M\left(\frac{-1+h}{2},\frac{6+p}{2},\frac{3+s}{2}\right)=M\left(\frac{6}{7},\frac{19}{7},\frac{32}{7}\right)$.

On comparing the respective coefficients, we obtain : $Q\left(\frac{19}{7}, -\frac{4}{7}, \frac{43}{7}\right)$ as the **image** of P in line.

35. $x+y+z=6000,\ x+3z=11000,\ x-2y+z=0$.

Let $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{bmatrix}, B=\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{vmatrix}=6\neq 0 \quad \therefore A^{-1}$ exists.

Consider C_{ij} be the cofactors of element a_{ij}.

$C_{11}=6,\quad C_{12}=2,\quad C_{13}=-2$

$C_{21}=-3,\quad C_{22}=0,\quad C_{23}=3$

$C_{31}=3,\quad C_{32}=-2,\quad C_{33}=-1$

$\therefore\ adj.A=\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}$

$\Rightarrow A^{-1}=\frac{1}{6}\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}$.

As $X=A^{-1}B$

$\Rightarrow X=\frac{1}{6}\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}=\frac{1}{6}\begin{bmatrix}3000\\ 12000\\ 21000\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}500\\ 2000\\ 3500\end{bmatrix}$

By equality of matrices, we get : $x=500,\ y=2000,\ z=3500$.

SECTION E

36. **(i)** As the cylinder is open at the top so, $2\pi rh+\pi r^2=75\pi\ cm^2$.

$\Rightarrow h=\frac{75\pi-\pi r^2}{2\pi r}=\frac{75-r^2}{2r}$

Now volume of cylinder, $V=\pi r^2 h=\pi r^2\times\frac{75-r^2}{2r}$

$\Rightarrow V=\frac{\pi(75r-r^3)}{2}$.

(ii) $\frac{dV}{dr}=\frac{\pi(75-3r^2)}{2}$.

(iii) As $\frac{dV}{dr}=\frac{\pi(75-3r^2)}{2}$ so, $\frac{d^2V}{dr^2}=\frac{\pi(0-3\times 2r)}{2}=-(3\pi r)$

For $\frac{dV}{dr}=0,\ \frac{\pi(75-3r^2)}{2}=0$

$\Rightarrow (75-3r^2)=0$

$\therefore r=5$ cm

Note that, $\left(\frac{d^2V}{dr^2}\right)_{\text{at } r=5}=-(15\pi)<0$

Clearly, V is maximum when $r=5$ cm.

OR

(iii) As $\frac{dV}{dr}=\frac{\pi(75-3r^2)}{2}$ so, $\frac{d^2V}{dr^2}=\frac{\pi(0-3\times 2r)}{2}=-(3\pi r)$

For $\frac{dV}{dr}=0,\ \frac{\pi(75-3r^2)}{2}=0$

$\Rightarrow (75-3r^2)=0$

$\Rightarrow r^2=\frac{75}{3}=25$

$\therefore r=5$ cm.

Note that, $\left(\frac{d^2V}{dr^2}\right)_{\text{at } r=5}=-(15\pi)<0$

Clearly, V is maximum when $r=5$ cm.

Now $h=\frac{75-r^2}{2r}=\frac{75-25}{10}=5$ cm.

That means, $h=r$.

Hence, the statement 'For maximum volume, $h>r$.' is false.

37. **(i)** $f(x)=-0.3x^2+kx+98.6$, being a polynomial function, is differentiable everywhere, hence, f is differentiable in $(0, 12)$.

(ii) $f'(x)=-0.6x+k$

Since $x=6$ is the critical point so, $f'(6)=0$

$\Rightarrow -0.6\times 6+k=0$

$\Rightarrow k=3.6$.

(iii) $f(x)=-0.3x^2+3.6x+98.6$

$\Rightarrow f'(x)=-0.6x+3.6=-0.6(x-6)$

In the Interval	$f'(x)$ is	Conclusion
$(0, 6)$	Positive	f is strictly increasing in (0, 6)
$(6, 12)$	Negative	f is strictly decreasing in (6, 12)

OR

(iii) $f(x)=-0.3x^2+3.6x+98.6$

$\Rightarrow f'(x)=-0.6x+3.6$ and $f''(x)=-0.6$.

For $f'(x)=0$, $-0.6x+3.6=0\ \therefore x=6$

As $f''(6) = -0.2 < 0$.

Hence, by second derivative test we can see that, $x = 6$ is a point of local maximum.

Also, the local maximum value $= f(6) = -0.3\times 6^2 + 3.6\times 6 + 98.6 = 109.4$.

38. **(i)** Let A : selected question is a multiple choice question.

Let B : the selected question is easy.

Clearly, we have a total of 1400 questions and out of which 900 are of multiple choice type questions.

$\therefore P(A) = \dfrac{900}{1400} = \dfrac{9}{14}$.

Note that there are 500 easy multiple choice type questions.

$\therefore P(A\cap B) = \dfrac{500}{1400} = \dfrac{5}{14}$.

($\because A\cap B =$ the question is an easy multiple choice question).

Therefore, $P(B|A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{\frac{5}{14}}{\frac{9}{14}} = \dfrac{5}{9}$.

(ii) Let C : the selected question is of True / False type.

$\because B\cap C =$ the question is an easy True / False type question.

$\therefore P(B\cap C) = \dfrac{300}{1400} = \dfrac{3}{14}$

Also, $P(C) = \dfrac{500}{1400} = \dfrac{5}{14}$.

Note that there are 500 True / False type questions.

Therefore, $P(B|C) = \dfrac{P(B\cap C)}{P(C)} = \dfrac{\frac{3}{14}}{\frac{5}{14}} = \dfrac{3}{5}$.

🕮 Detailed Solutions (PTS-24)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. We have $\cos^{-1}x > \sin^{-1}x$

$$\Rightarrow \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x$$

$$\Rightarrow \frac{\pi}{2} > 2\sin^{-1}x$$

$$\Rightarrow \frac{\pi}{4} > \sin^{-1}x$$

$$\Rightarrow \sin\sin^{-1}x < \sin\frac{\pi}{4}$$

$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

$$\therefore x \in \left[-1, \frac{1}{\sqrt{2}}\right).$$

OR

As $a^2 \geq 0$ i.e., $1 + a^2 > 0$ i.e., $1 + a.a > 0$ i.e., $(a, a) \in R$.

$\therefore$ R is reflexive.

Let $(a, b) \in R$ and $(b, c) \in R$.

Put $a = 1,\ b = -\frac{1}{2},\ c = -1$.

Note that $\left(1, -\frac{1}{2}\right) \in R$ as $1 + 1\left(-\frac{1}{2}\right) = \frac{1}{2} > 0$.

Similarly, $\left(-\frac{1}{2}, -1\right) \in R$ as $1 + \left(-\frac{1}{2}\right)(-1) = \frac{3}{2} > 0$

But $(1, -1) \notin R$ as $1 + (1)(-1) = 0 > 0$.

Hence, R is not transitive.

22. $f(x) = \left[x(x-1)+1\right]^{\frac{1}{3}},\ 0 \leq x \leq 1$

$$\Rightarrow f'(x) = \frac{2x-1}{3(x^2 - x + 1)^{2/3}}$$

For $f'(x) = \dfrac{2x-1}{3(x^2 - x + 1)^{2/3}} = 0 \quad \Rightarrow x = \dfrac{1}{2} \in [0, 1]$

So, $f(0)=\left[0(0-1)+1\right]^{\frac{1}{3}}=1$, $f\left(\frac{1}{2}\right)=\left[\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)+1\right]^{\frac{1}{3}}=\left(\frac{3}{4}\right)^{1/3}$, $f(1)=\left[1(1-1)+1\right]^{\frac{1}{3}}=1$.

Hence, the absolute maximum value of $f(x)$ is '1'.

23. Re-writing the given line, we get $\dfrac{3\left(x+\frac{2}{3}\right)}{1}=\dfrac{-2\left(y-\frac{1}{2}\right)}{1}=\dfrac{z-(-5)}{1}$

i.e., $\dfrac{x-\left(-\frac{2}{3}\right)}{1/3}=\dfrac{y-\frac{1}{2}}{-1/2}=\dfrac{z-(-5)}{1}$

Therefore, the d.r.'s of given line are $\frac{1}{3},-\frac{1}{2},1$ i.e., $2,-3,6$.

Required eq. of line is : $\dfrac{x-(-1)}{2}=\dfrac{y-(-2)}{-3}=\dfrac{z-0}{6}$ i.e., $\dfrac{x+1}{2}=\dfrac{y+2}{-3}=\dfrac{z}{6}$.

OR

$4x=3y=2z$ can be rewritten as $\dfrac{4x}{12}=\dfrac{3y}{12}=\dfrac{2z}{12}$ i.e., $\dfrac{x}{3}=\dfrac{y}{4}=\dfrac{z}{6}$.

Therefore, the d.r.'s of line are 3, 4, 6.

The direction ratios of x-axis are 1, 0, 0.

So the required angle θ between the line and x-axis is, $\cos\theta=\dfrac{\left|3\times1+4\times0+6\times0\right|}{\sqrt{3^2+4^2+6^2}\sqrt{1^2+0^2+0^2}}$

$\Rightarrow \theta=\cos^{-1}\left(\dfrac{3}{\sqrt{61}}\right)$.

24. Here $f\left(-\frac{\pi}{6}\right)=k$.

Also, $\lim\limits_{x\to-\frac{\pi}{6}}\dfrac{\sqrt{3}\sin x+\cos x}{x+\frac{\pi}{6}}=\lim\limits_{x+\frac{\pi}{6}\to0}\dfrac{2\sin\left(x+\frac{\pi}{6}\right)}{x+\frac{\pi}{6}}=2\times1=2$

For the continuity of $f(x)$ at $x=-\frac{\pi}{6}$, we have $f\left(-\frac{\pi}{6}\right)=\lim\limits_{x+\frac{\pi}{6}\to0}f(x)$

$\therefore k=2$.

25. Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

Consider $(\vec{a}+\vec{b}+\vec{c})\times\vec{b}=\vec{0}\times\vec{b}$

$\Rightarrow \vec{a}\times\vec{b}+\vec{b}\times\vec{b}+\vec{c}\times\vec{b}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}+\vec{0}-\vec{b}\times\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}$...(i)

Also $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}\times\vec{c}$

$\Rightarrow \vec{a}\times\vec{c}+\vec{b}\times\vec{c}+\vec{c}\times\vec{c}=\vec{0}$

$\Rightarrow -\vec{c}\times\vec{a}+\vec{b}\times\vec{c}+\vec{0}=\vec{0}$

$\Rightarrow \vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ ...(ii)

By (i) and (ii), $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

SECTION C

26. Put $2x = y \Rightarrow dx = \frac{dy}{2}$

So, $\int\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx = \frac{1}{2}\int\left[\frac{2}{y}-\frac{2}{y^2}\right]e^{y}dy$

$\Rightarrow \quad = \int\left[\frac{1}{y}+\frac{-1}{y^2}\right]e^{y}dy$

$\Rightarrow \quad = \frac{1}{y}\times e^{y} + C$ $\qquad \left\{\because \int\left[f(x)+f'(x)\right]e^{x}dx = f(x)\times e^{x} + C\right.$

$\Rightarrow \quad = \frac{e^{2x}}{2x} + C.$

27. Let E : the three cards drawn are all spades, E_1 : the lost card is a spade and, E_2 : the lost card is a non-spade.

Here $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{39}{52} = \frac{3}{4}$,

$P(E|E_1) = \frac{^{12}C_3}{^{51}C_3} = \frac{12\times 11\times 10}{51\times 50\times 49}$, $P(E|E_2) = \frac{^{13}C_3}{^{51}C_3} = \frac{13\times 12\times 11}{51\times 50\times 49}$

By Bayes' Theorem, $P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)}$

$\Rightarrow P(E_1|E) = \frac{\frac{12\times 11\times 10}{51\times 50\times 49}\times\frac{1}{4}}{\frac{12\times 11\times 10}{51\times 50\times 49}\times\frac{1}{4}+\frac{13\times 12\times 11}{51\times 50\times 49}\times\frac{3}{4}}$

$\Rightarrow P(E_1|E) = \frac{10}{10+13\times 3}$

$\therefore P(E_1|E) = \frac{10}{49}.$

OR

We have $P(A) = 0.70$, $P(B) = 0.80$. So, $P(\bar{A}) = 1 - P(A) = 0.30$, $P(\bar{B}) = 1 - P(B) = 0.20$.

(i) $P(A\cap B) = P(A)\times P(B) = 0.70\times 0.80 = 0.56$

(ii) $P(A\cup B) = P(A)+P(B)-P(A\cap B) = 0.70+0.80-0.56 = 0.94$

(iii) $P(\text{exactly one mission succeeds}) = P(A)\times P(\bar{B}) + P(\bar{A})\times P(B)$

$\Rightarrow P(\text{exactly one mission succeeds}) = 0.70\times 0.20 + 0.30\times 0.80 = 0.14 + 0.24 = 0.38$.

28. Put $x^2 = y$ to make partial fractions.

$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)} = \frac{A}{y+2} + \frac{B}{y+3}$

$\Rightarrow y+1 = A(y+3)+B(y+2) \qquad \ldots(i)$

Comparing coefficients of y and constant terms on both sides of (i), we get

$A+B=1$ and $3A+2B=1$

On solving these equations, we get $A=-1, B=2$

Now, $\int\frac{x^2+1}{(x^2+2)(x^2+3)}dx = \int\frac{-1}{x^2+2}dx + 2\int\frac{1}{x^2+3}dx$

$$=-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C.$$

OR

Let $I=\int\frac{\sin x-x\cos x}{x(x+\sin x)}dx$

$\Rightarrow I=\int\frac{x+\sin x-x-x\cos x}{x(x+\sin x)}dx$

$\Rightarrow I=\int\left(\frac{x+\sin x}{x(x+\sin x)}-\frac{x+x\cos x}{x(x+\sin x)}\right)dx$

$\Rightarrow I=\int\frac{1}{x}dx-\int\frac{1+\cos x}{x+\sin x}dx$ (Put $x+\sin x=t\Rightarrow(1+\cos x)dx=dt$ in 2^{nd} integral

$\therefore I=\log|x|-\int\frac{dt}{t}$

$\Rightarrow I=\log|x|-\log|t|+C$

$\therefore I=\log|x|-\log|x+\sin x|+C.$

29. We have $\frac{dy}{dx}+2xy=y$

$\Rightarrow\frac{dy}{dx}=y(1-2x)$

$\Rightarrow\int\frac{dy}{y}=\int(1-2x)dx$

$\Rightarrow\log y=x-x^2+k$

As $y=1,\ x=0$ so, $\log 1=0-0^2+k\Rightarrow k=0$

That implies, $\log y=x-x^2$

$\Rightarrow y=e^{x-x^2}$

Also, at $x=2$, $y=e^{2-2^2}=e^{-2}$ i.e., $y=f(2)=e^{-2}$.

OR

$y+\frac{d}{dx}(xy)=x\log x$

$\Rightarrow y+x\frac{dy}{dx}+y\frac{d}{dx}(x)=x\log x$

$\Rightarrow y+x\frac{dy}{dx}+y(1)=x\log x$

$\therefore\frac{dy}{dx}+\frac{2}{x}y=\log x$

On comparing with $\frac{dy}{dx}+P(x)y=Q(x)$, we get $P(x)=\frac{2}{x}$, $Q(x)=\log x$.

Now, $I.F.=e^{\int\frac{2}{x}dx}=e^{\log x^2}=x^2$

So solution is given by $y(x^2)=\int x^2\log x\,dx+C$

Consider $I=\int x^2\log x\,dx$ (using integral By Parts

$\Rightarrow \quad = \log x \int x^2 dx - \int \left[\frac{d}{dx} \log x \int x^2 dx \right] dx$

$\Rightarrow \quad = \frac{x^3}{3} \log x - \frac{1}{3} \int x^2 dx$

$\Rightarrow \quad = \frac{x^3}{3} \log x - \frac{1}{9} x^3$

So the required solution is, $y(x^2) = \frac{x^3}{3} \log x - \frac{1}{9} x^3 + C$

That is, $y = \frac{x \log x}{3} - \frac{x}{9} + C x^{-2}$.

30. Let $I = \int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx \quad \ldots(i)$

Using $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, we have : $I = \int_0^\pi \frac{e^{\cos(\pi - x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}} dx$

$\Rightarrow I = \int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx \quad \ldots(ii)$

Adding (i) and (ii), we get : $2I = \int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx + \int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx$

$\Rightarrow 2I = \int_0^\pi \frac{e^{\cos x} + e^{-\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

$\Rightarrow 2I = \int_0^\pi 1\, dx$

$\Rightarrow I = \frac{1}{2} [x]_0^\pi$

$\Rightarrow I = \frac{1}{2} [\pi - 0]$

$\therefore \; I = \frac{\pi}{2}$.

31. **(i)** To maximize $z = 8x + 9y$

Corner Points	Value of z
A(0,1)	9
$B\left(\frac{3}{2}, 1\right)$	21
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$\frac{294}{13} = 22\frac{8}{13} \leftarrow$ Max. value
D(2,0)	16
O(0, 0)	0

Therefore, the maximum value of z is $22\frac{8}{13}$.

(ii) Maximum value of z is attained at $\left(\frac{30}{13}, \frac{6}{13}\right)$.

(iii) Possible constraints for the given feasible region are

$x \geq 0,\ y \geq 0,\ y \leq 1,\ 2x+3y \leq 6,\ 3x-2y \leq 6.$

SECTION D

32. Re-writing the given curve as, $\frac{x^2}{9}+\frac{y^2}{4}=1$...(i)

Curve (i) represents an ellipse of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose major axis is along x-axis.

So, required area $=\int_0^3 y_1 dx$

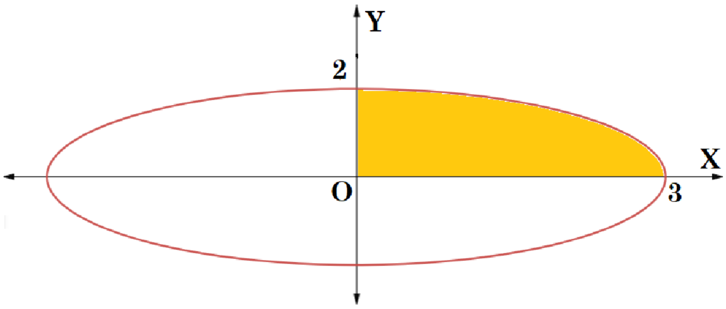
$\Rightarrow \quad =\frac{2}{3}\int_0^3 \sqrt{3^2-x^2}\,dx$

$\Rightarrow \quad =\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$

$\Rightarrow \quad =\frac{2}{3}\left[0+\frac{9}{2}\sin^{-1}1\right]-[0+0]$

$\Rightarrow \quad =\frac{2}{3}\times\frac{9\pi}{4}$

$\Rightarrow \quad =\frac{3\pi}{2}$ Sq.units.



OR

We have $y=|x+2|+|x-2|$, if $-3 \leq x \leq 3$

$$\Rightarrow y=\begin{cases}-x-2-x+2=-2x, \text{if } -3 \leq x \leq -2\\ x+2-x+2=4, \text{if } -2 \leq x \leq 2\\ x+2+x-2=2x, \text{if } 2 \leq x \leq 3\end{cases}$$

So, required area $=\int_{-3}^{3} y dx$

$\Rightarrow \quad =\int_{-3}^{-2}-2x dx+\int_{-2}^{2}4 dx+\int_{2}^{3}2x dx$

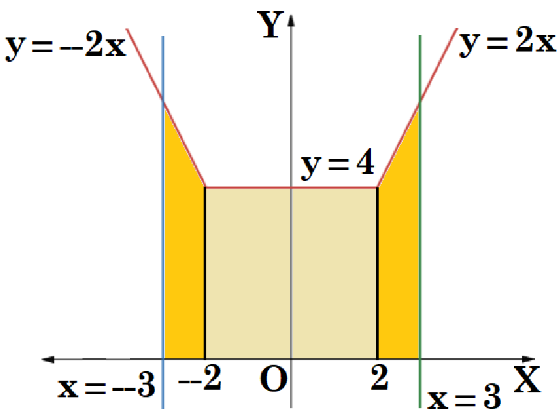
$\Rightarrow \quad =-2\left[\frac{x^2}{2}\right]_{-3}^{-2}+4[x]_{-2}^{2}+2\left[\frac{x^2}{2}\right]_2^3$

$\Rightarrow \quad =-\left[x^2\right]_{-3}^{-2}+4[x]_{-2}^{2}+\left[x^2\right]_2^3$

$\Rightarrow \quad =-[4-9]+4[2-(-2)]+[9-4]$

$\Rightarrow \quad =5+16+5$

$\Rightarrow \quad =26$ Sq. units.



33. We have $(a,b)\,R\,(c,d)$ given as $ad(b+c)=bc(a+d)$

Reflexivity : Let $(a,b) \in N \times N$

$\Rightarrow ab(b+a)=ba(a+b)$

$\therefore (a,b)\,R\,(a,b)$.

So, R is reflexive.

Symmetry : Let $(a,b),\ (c,d) \in N \times N$

Let $(a,b)\,R\,(c,d) \Leftrightarrow ad(b+c)=bc(a+d)$

$\Rightarrow cb(d+a) = da(c+b)$

$\therefore (c,d)\,R\,(a,b)$.

So, R is symmetric.

Transitivity : Let $(a,b), (c,d), (e,f) \in N \times N$

Since $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$

$\Rightarrow \frac{1}{b}+\frac{1}{c}=\frac{1}{a}+\frac{1}{d}$ (Dividing by 'abcd' both sides)

That is, $(a,b)R(c,d) \Leftrightarrow \frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$ and $(c,d)R(e,f) \Leftrightarrow \frac{1}{c}+\frac{1}{f}=\frac{1}{d}+\frac{1}{e}$

Adding these two equations, $\frac{1}{a}+\frac{1}{d}+\frac{1}{c}+\frac{1}{f}=\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$

$\Rightarrow \frac{1}{a}+\frac{1}{f}=\frac{1}{b}+\frac{1}{e}$

$\therefore (a,b)\,R\,(e,f)$

Hence, R is transitive.

Since R is reflexive, symmetric and transitive so, it is an equivalence relation.

34. (i) Since $\vec{a}.\left(\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}\right)=1$ i.e., $\vec{a}.(\vec{b}+\vec{c}) = |\vec{b}+\vec{c}|$

$\Rightarrow (\hat{i}+\hat{j}+\hat{k}).\{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}\} = \sqrt{(2+\lambda)^2+6^2+(-2)^2}$

$\Rightarrow 2+\lambda+6-2 = \sqrt{\lambda^2+4\lambda+4+40}$

$\Rightarrow \lambda^2+12\lambda+36 = \lambda^2+4\lambda+44$

$\therefore \lambda = 1$.

(ii) Given that $\vec{c} = \hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow \hat{c} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{1+4+9}} = \frac{\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{3\hat{k}}{\sqrt{14}}$ …(i)

Comparing (i) with $\cos\alpha\,\hat{i}+\cos\beta\,\hat{j}+\cos\gamma\,\hat{k}$, we get : $\cos\beta = \frac{2}{\sqrt{14}}$ i.e., $\beta = \cos^{-1}\frac{2}{\sqrt{14}}$.

OR

We have

$\vec{a}_1 = 3\hat{i}+2\hat{j}-4\hat{k}$, $\vec{b}_1 = \hat{i}+2\hat{j}+2\hat{k}$;

$\vec{a}_2 = 5\hat{i}-2\hat{j}$, $\vec{b}_2 = 3\hat{i}+2\hat{j}+6\hat{k}$

$\Rightarrow \vec{a}_2-\vec{a}_1 = 2\hat{i}-4\hat{j}+4\hat{k}$.

And, $\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 3 & 2 & 6\end{vmatrix} = \hat{i}(12-4)-\hat{j}(6-6)+\hat{k}(2-6)$

$\Rightarrow \vec{b}_1\times\vec{b}_2 = 8\hat{i}-4\hat{k}$

$\because \left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right) = 16-16 = 0$

As S.D. $= \frac{\left|\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$\Rightarrow$ S.D. $= 0$.

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection, we have $3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$

$\Rightarrow 3+\lambda=5+3\mu$ ...(i)

$\Rightarrow 2+2\lambda=-2+2\mu$ ...(ii)

$\Rightarrow -4+2\lambda=6\mu$ ...(iii)

Solving (i) and (ii) we get, $\mu=-2$ and $\lambda=-4$.

Substituting in equation of line we get

$\vec{r}=5\hat{i}-2\hat{j}+(-2)(3\hat{i}+2\hat{j}+6\hat{k})=-\hat{i}-6\hat{j}-12\hat{k}$

So, the point of intersection is $(-1,-6,-12)$.

35. Let $A=\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}$, $B=\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}$

$\therefore AB=\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}$

$\Rightarrow AB=8\,I_3$ ...(i)

Consider the given systems of equations : $x-y+z=4$, $x-2y-2z=9$, $2x+y+3z=1$.

These equations can be expressed as : $BX=D$ where $B=\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $C=\begin{bmatrix}4\\9\\1\end{bmatrix}$

Therefore, $X=B^{-1}C=\frac{1}{8}AC$ $\left[\text{By (i), } AB=8\,I \Rightarrow \left(\frac{1}{8}A\right)B=I \;\therefore B^{-1}=\frac{1}{8}A\right.$

So, $X=\frac{1}{8}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}$ $\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\-2\\-1\end{bmatrix}$

$\therefore$ By equality of matrices : $x=3, y=-2, z=-1$.

SECTION E

36. **(i)** Clearly, the area of the open tank will be same as the area of metal sheet.

That is, area of metal sheet $(A)=x\times x+4(x\times h)=x^2+4hx$.

(ii) Volume of the water stored in the tank is, $4000=x\times x\times h$ i.e., $4000=x^2h$.

(iii) $A=x^2+4hx$

$\Rightarrow A=x^2+4\times\frac{4000}{x}$

$\therefore \frac{dA}{dx}=2x-\frac{16000}{x^2}$, $\frac{d^2A}{dx^2}=2+\frac{32000}{x^3}$

For $\frac{dA}{dx}=0$, $2x-\frac{16000}{x^2}=0$

$\Rightarrow x=20$ m.

Also, $4000=x^2h$

$\Rightarrow 4000=20^2\times h$ $\therefore$ $h=10$ m.

OR

(iii) $A = x^2 + 4hx$

$\Rightarrow A = x^2 + 4\times\frac{4000}{x}$

$\therefore\ \frac{dA}{dx} = 2x - \frac{16000}{x^2},\ \frac{d^2A}{dx^2} = 2 + \frac{32000}{x^3}$

For $\frac{dA}{dx} = 0,\ 2x - \frac{16000}{x^2} = 0$

$\Rightarrow x = 20$ m.

Note that $\left.\frac{d^2A}{dx^2}\right]_{\text{at } x=20\text{ m}} = 2 + \frac{32000}{8000} = 6 > 0$ so, A is least when $x = 20$ m.

Now the least value of $A = 20^2 + 4\times\frac{4000}{20}$ (Putting $x = 20$ m in $A = x^2 + 4\times\frac{4000}{x}$

$\Rightarrow A = 400 + 4\times 200$

$\therefore A = 1200\ \text{m}^2$.

37. **(i)** Here $P(E_1) = P(E_2) = \frac{1}{2}$ and, $P(A\,|\,E_1) = \frac{3}{9} = \frac{1}{3}$.

(ii) Note that $P(A\,|\,E_2) = \frac{5}{5+n}$.

By Bayes' theorem, $P(E_2\,|\,A) = \frac{P(E_2)P(A\,|\,E_2)}{P(E_1)P(A\,|\,E_1) + P(E_2)P(A\,|\,E_2)}$

$$\Rightarrow \frac{3}{5} = \frac{\frac{1}{2}\times\frac{5}{5+n}}{\frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{5}{5+n}} = \frac{15}{20+n}$$

$\Rightarrow 25 = 20 + n$

$\therefore n = 5$.

(iii) $P(A) = P(E_1)\times P(A\,|\,E_1) + P(E_2)\times P(A\,|\,E_2)$

So, $P(A) = \frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{5}{10} = \frac{5}{12}$.

OR

(iii) By Bayes' theorem, $P(E_1\,|\,A) = \frac{P(E_1)\times P(A\,|\,E_1)}{P(E_1)\times P(A\,|\,E_1) + P(E_2)\times P(A\,|\,E_2)}$

$$\Rightarrow P(E_1\,|\,A) = \frac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{5}{10}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{2}} = \frac{1}{1 + \frac{3}{2}} = \frac{2}{5}.$$

Alternatively, $P(E_1\,|\,A) = 1 - P(E_2\,|\,A) = 1 - \frac{3}{5} = \frac{2}{5}$.

38. **(i)** Given that volume is, $125\pi = \pi r^2 h$

$\Rightarrow h = \frac{125}{r^2}$ ...(i)

Now surface area, $S = 2\pi r h + \pi r^2$

$\Rightarrow S = 2\pi \times \frac{125}{r} + \pi r^2$ [By (i)

$\therefore S = \frac{250\pi}{r} + \pi r^2$

Also, $S = 2\pi \times \frac{5\sqrt{5}}{\sqrt{h}} \times h + \pi \times \frac{125}{h}$ [By (i), $r^2 = \frac{125}{h}$, $r = \frac{5\sqrt{5}}{\sqrt{h}}$

$\therefore S = 10\pi\sqrt{5}\ h + \frac{125\pi}{h}$

(ii) For $S = \frac{250\pi}{r} + \pi r^2$, $\frac{dS}{dr} = 2\pi \times \left(-\frac{125}{r^2}\right) + 2\pi r$

For $\frac{dS}{dr} = 0$, $2\pi \times \left(-\frac{125}{r^2}\right) + 2\pi r = 0 \quad \Rightarrow r = 5$

Also $\frac{d^2S}{dr^2} = 2\pi \times \left(\frac{250}{r^3}\right) + 2\pi$

$\because \left.\frac{d^2S}{dr^2}\right]_{\text{at } r=5} = 2\pi \times \left(\frac{250}{125}\right) + 2\pi = 6\pi > 0$ so, S is least at $r = 5$.

Therefore the radius is, $r = 5$ cm and height is, $h = \frac{125}{5^2} = 5$ cm.

Clearly, $r = h$.

Detailed Solutions (PTS-25)

SECTION A

For the **Detailed Solutions of all MCQs from Section A (Q01-20)**, please refer our YouTube Channel - **YouTube.com/@theopgupta**

Or, simply Click on the PTS Logo given here.

SECTION B

21. Let $\theta = \cos^{-1} x$. Then $x = \cos\theta$.

Also, for all $x \in \left[\frac{1}{2}, 1\right]$, $\theta \in \left[0, \frac{\pi}{3}\right]$.

Let $y = \cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3}}{2}\sqrt{1-\cos^2\theta}\right]$

$\Rightarrow y = \theta + \cos^{-1}\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] = \theta + \cos^{-1}\left[\cos\left(\theta - \frac{\pi}{3}\right)\right]$

$\left[\because 0 \le \theta \le \frac{\pi}{3} \quad \Rightarrow -\frac{\pi}{3} \le \theta - \frac{\pi}{3} \le 0 \quad \Rightarrow 0 \le \frac{\pi}{3} - \theta \le \frac{\pi}{3}\right]$

$\therefore y = \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \theta\right)\right] = \theta + \frac{\pi}{3} - \theta = \frac{\pi}{3}$.

Note that $\cos(-A) = \cos A$.

OR

As $\sin^{-1} x$ and, $\cos^{-1} x$ is defined for $x \in [-1, 1]$.

Also $\tan^{-1} x$ is defined for $x \in \mathbb{R}$.

Clearly, domain of $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is $x \in [-1, 1]$.

Note that, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \ \forall\ x \in [-1, 1]$.

As $-1 \le x \le 1$

$\Rightarrow \tan^{-1}(-1) \le \tan^{-1} x \le \tan^{-1}(1)$

$\Rightarrow -\frac{\pi}{4} \le \tan^{-1} x \le \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{4} + \frac{\pi}{2} \le \frac{\pi}{2} + \tan^{-1} x \le \frac{\pi}{4} + \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{4} \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le \frac{3\pi}{4}$

$\Rightarrow \frac{\pi}{4} \le f(x) \le \frac{3\pi}{4}$

So, the range of $f(x)$ is $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

22. We have $f(x)=x-\log\left(\frac{x+1}{x}\right)$, $x>0$

$\Rightarrow f(x)=x-\log(x+1)+\log x$

$\Rightarrow f'(x)=1-\frac{1}{x+1}+\frac{1}{x}$

$\Rightarrow f'(x)=\frac{x^2+x+1}{x^2+x}$

$\Rightarrow f'(x)=1+\frac{1}{x^2+x}$

$\because\ f'(x)=\frac{x^2+x+1}{x^2+x}>0\ \forall x\in(0,\infty)$

$\therefore f(x)$ is increasing for all $x>0$, i.e. $x\in(0,\infty)$.

23. Let $L_1: x=py+q,\ z=ry+s$ $\Rightarrow \frac{x-q}{p}=\frac{y-0}{1}=\frac{z-s}{r}$ $\therefore$ The d.r.'s are $p, 1, r$.

Let $L_2: x=p'y+q',\ z=r'y+s'$ $\Rightarrow \frac{x-q'}{p'}=\frac{y-0}{1}=\frac{z-s'}{r'}$ $\therefore$ The d.r.'s are $p', 1, r'$.

Using $a_1a_2+b_1b_2+c_1c_2=0$ for perpendicular lines, we have : $p.p'+1.1+r.r'=0$

$\Rightarrow pp'+rr'+1=0$.

OR

$\because\ \left|\hat{a}+\hat{b}\right|^2=1$

$\Rightarrow |\hat{a}|^2+\left|\hat{b}\right|^2+2\hat{a}.\hat{b}=1$

$\Rightarrow 2\hat{a}.\hat{b}=1-1-1$

$\Rightarrow |\hat{a}|\left|\hat{b}\right|\cos\theta=-\frac{1}{2}$

$\Rightarrow 1\times1\times\cos\theta=-\frac{1}{2}$

$\Rightarrow \cos\theta=-\frac{1}{2}$

$\therefore\ \theta=\frac{2\pi}{3}$.

24. $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\cot^{-1}a$

$\Rightarrow \cos^{-1}\left(\frac{1-\frac{y^2}{x^2}}{1+\frac{y^2}{x^2}}\right)=\cot^{-1}a$

$\Rightarrow \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)=\cot^{-1}a$ [Put $\frac{y}{x}=\tan\theta$

$\Rightarrow \cos^{-1}(\cos 2\theta)=\cot^{-1}a$ $\Rightarrow 2\theta=\cot^{-1}a$

$\Rightarrow 2\tan^{-1}\left(\frac{y}{x}\right)=\cot^{-1}a$

$\Rightarrow \dfrac{y}{x} = \tan\left(\dfrac{1}{2}\cot^{-1} a\right)$

$\therefore \dfrac{x \times y' - y \times 1}{x^2} = 0$

$\Rightarrow x \times y' - y = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

$\therefore \dfrac{d^2y}{dx^2} = -\dfrac{y}{x^2} + \dfrac{1}{x} \times \dfrac{dy}{dx} = -\dfrac{y}{x^2} + \dfrac{1}{x} \times \dfrac{y}{x} = 0$ $\left[\because \dfrac{dy}{dx} = \dfrac{y}{x}.\right.$

25. As $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$ so, $(2\hat{i} + 6\hat{j} + 27\hat{k}) \,||\, (\hat{i} + p\hat{j} + q\hat{k})$

Therefore, $\dfrac{2}{1} = \dfrac{6}{p} = \dfrac{27}{q}$ ($\because$ The d.r.'s of parallel vectors are proportional

Consider $\dfrac{2}{1} = \dfrac{6}{p}$ and $\dfrac{2}{1} = \dfrac{27}{q}$

$\therefore p = 3,\ q = \dfrac{27}{2}$

Therefore, $p + q = 3 + \dfrac{27}{2} = \dfrac{33}{2}$.

SECTION C

26. Let $I = \int_{-\pi}^{\pi} (\cos ax - \sin ax)^2 dx$

$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 ax - 2\cos ax \sin ax)dx$

$\Rightarrow I = \int_{-\pi}^{\pi} (1 - \sin 2ax)dx$

$\Rightarrow I = \int_{-\pi}^{\pi} 1 dx - \int_{-\pi}^{\pi} \sin 2ax\, dx$

$\because f(x) = 1 = f(-x)$

and, $g(x) = \sin 2ax$

$\Rightarrow g(-x) = \sin 2a(-x) = -\sin 2ax = -g(x)$

$\therefore f(x)$ is even function, and $g(x)$ is odd function.

Using $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{ if } f \text{ is even} \\ 0, \text{ if } f \text{ is odd} \end{cases}$, we get $I = 2\int_0^{\pi} 1\,dx - 0$

Therefore, $I = 2[x]_0^{\pi}$

$\therefore I = 2\pi$.

27. Let E_1, E_2 and E_3 be the events that the bag I, bag II and bag III is chosen respectively.
Let E : the two balls drawn from the chosen bag are white and red.

Now $P(E_1) = P(E_2) = P(E_3) = \dfrac{1}{3}$, $P(E|E_1) = \dfrac{1}{6} \times \dfrac{3}{6} \times 2$, $P(E|E_2) = \dfrac{2}{4} \times \dfrac{1}{4} \times 2$, $P(E|E_3) = \dfrac{4}{9} \times \dfrac{2}{9} \times 2$

By Bayes' theorem, $P(E_3 | E) = \dfrac{P(E|E_3)\ P(E_3)}{\sum_{i=1}^{i=3}[P(E|E_i)\ P(E_i)]}$

$\Rightarrow P(E_3 | E) = \dfrac{\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}{\frac{1}{3}\times\frac{1}{6}\times\frac{3}{6}\times 2+\frac{1}{3}\times\frac{2}{4}\times\frac{1}{4}\times 2+\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}$

$\therefore\ P(E_3 | E) = \dfrac{64}{199}$.

OR

We have $P(M) = 40\%,\ P(F) = 60\%,\ P(X|M) = 70\%,\ P(X|F) = 50\%$.

Therefore, overall probability that a randomly chosen voter supports candidate X is given by, $P(X) = P(X|M)\times P(M) + P(X|F)\times P(F)$

$\Rightarrow P(X) = 0.70\times 0.40 + 0.50\times 0.60 = 0.28 + 0.30 = 0.58$

Now, the Probability that a voter does not support $X = 1 - P(X) = 1 - 0.58 = 0.42$.

28. Let $I = \int \dfrac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx = \int \dfrac{1+x+\sqrt{x}\sqrt{1+x}}{\sqrt{x}+\sqrt{1+x}}dx$

$\Rightarrow I = \int \sqrt{1+x}\times\left(\dfrac{\sqrt{1+x}+\sqrt{x}}{\sqrt{x}+\sqrt{1+x}}\right)dx$

$\Rightarrow I = \int \sqrt{1+x}dx = \dfrac{2}{3}(1+x)^{3/2} + C$

That is, $I = \dfrac{2}{3}(1+x)\sqrt{1+x} + C$.

OR

Let $I = \int \dfrac{x^3 dx}{\sqrt{1+x^2}} = \dfrac{1}{2}\int \dfrac{2x.x^2}{\sqrt{1+x^2}}dx$

Put $1+x^2 = t^2 \Rightarrow 2x\,dx = 2t\,dt$

$\Rightarrow I = \dfrac{1}{2}\int \dfrac{2t.(t^2-1)}{\sqrt{t^2}}dt = \int (t^2-1)\,dt = \dfrac{t^3}{3} - t + C$

$\therefore I = \dfrac{(1+x^2)^{\frac{3}{2}}}{3} - \sqrt{1+x^2} + C$.

29. We have $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$ $\qquad \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x} - \tan\left(\dfrac{y}{x}\right)$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore v + x\dfrac{dv}{dx} = v - \tan v$

$\Rightarrow \int \cot v dv = -\int \dfrac{dx}{x}$

$\Rightarrow \log|\sin v| = -\log|x| + \log|C|$

$\Rightarrow \log|\sin v| = \log\left|\dfrac{C}{x}\right|$

$\therefore \sin\left(\frac{y}{x}\right) = \frac{C}{x}$ or, $x\sin\left(\frac{y}{x}\right) = C$.

OR

We have $\frac{dy}{dx} + 3y\cot x = \sin 2x$.

It is linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$.

Here $P(x) = 3\cot x, Q(x) = \sin 2x$.

So, integrating factor $= e^{\int 3\cot x dx} = e^{3\log \sin x} = \sin^3 x$

$\therefore$ The solution is : $y\sin^3 x = \int \sin^3 x \sin 2x dx + C$

$\Rightarrow y\sin^3 x = 2\int \sin^4 x \cos x\, dx + C$

$\Rightarrow y\sin^3 x = 2 \times \frac{\sin^5 x}{5} + C$

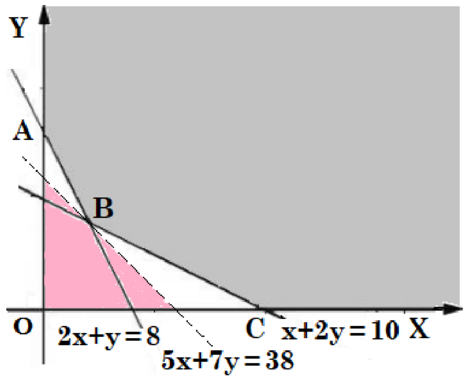
$\Rightarrow y\sin^3 x = \frac{2}{5}\sin^5 x + C$.

It is given $y = 2$ when $x = \frac{\pi}{2}$ so, $2\sin^3 \frac{\pi}{2} = \frac{2}{5}\sin^5 \frac{\pi}{2} + C \quad \Rightarrow C = \frac{8}{5}$

Hence required solution is $y\sin^3 x = \frac{2}{5}\sin^5 x + \frac{8}{5}$ or, $5y = 2\sin^2 x + 8\,\text{cosec}^3 x$.

30. Consider the graph shown.

Corner Points	Value of Z
A(0, 8)	560
B(2, 4)	380 ← Min. value
C(10, 0)	500



Since the feasible region is unbounded so,
380 may or may not be minimum value of Z.

To check, draw $50x + 70y < 380$ i.e., $5x + 7y < 38$.

As in the half plane $5x + 7y < 38$, there is no point common with the feasible region.

Hence, minimum value of Z is 380.

31. Let $I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx \quad \ldots(i)$

$\Rightarrow \quad = \int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{8+2-x} + \sqrt{10-(8+2-x)}} dx$

$\Rightarrow \quad = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x} + \sqrt{x}} dx \ldots(ii)$

Adding (i) and (ii), we get : $2I = \int_2^8 1\, dx$

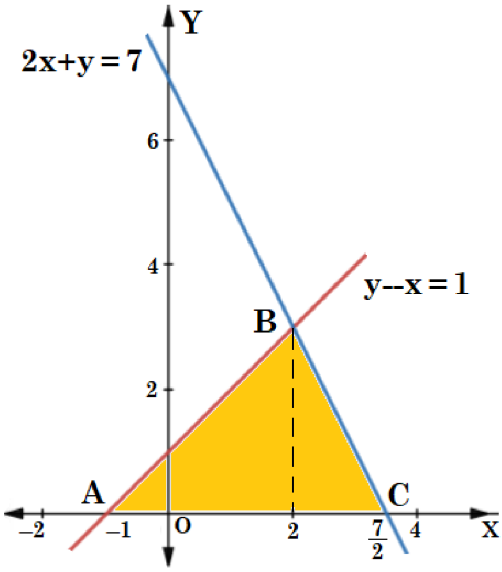
$\Rightarrow 2I = [x]_2^8 = 8 - 2$

$\Rightarrow 2I = 6$

$\therefore I = 3$.

SECTION D

32. Let the given lines form ΔABC, where
$AB: y - x = 1$ ...(i)
and, $BC: 2x + y = 7$ ...(ii)



Solving (i) and (ii), we get B(2, 3).

Also we have A(–1, 0), $C\left(\frac{7}{2}, 0\right)$.

Consider the figure shown here.

$\therefore$ Required area $= \int_{-1}^{2} y_{AB}dx + \int_{2}^{7/2} y_{BC}dx$

$\Rightarrow \quad = \int_{-1}^{2}(x+1)dx + \int_{2}^{7/2}(7-2x)dx$

$\Rightarrow \quad = \left[\frac{(x+1)^2}{2}\right]_{-1}^{2} + \left[\frac{(7-2x)^2}{2(-2)}\right]_{2}^{7/2}$

$\Rightarrow \quad = \frac{1}{2}[9-0] - \frac{1}{4}[0-9]$

$\Rightarrow \quad = \frac{27}{4}$ Sq. units.

33. $f(x) = \frac{x-1}{x-2}$

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$.

That is, $\frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$

$\Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one function.

Let $y = f(x),\ y \in B$

That is, $y = \frac{x-1}{x-2} \quad \Rightarrow xy - 2y = x - 1$

$\Rightarrow xy - x = 2y - 1 \quad \Rightarrow x = \frac{2y-1}{y-1}$

Clearly, $y \in B$ for all $x \in A$. That implies, Range = Codomain.

So, f is onto.

OR

$S = \{(a,b) : a, b \in A,\ |a-b|$ is divisible by $3\}$ is defined on the set $A = \{x \in Z : 0 \le x \le 12\}$.

Clearly, S is reflexive, as 3 divides $|a-a| = 0$, for all a ∈ A.

Further if (a, b) ∈ S, then 3 divides $|a-b|$. Therefore, 3 divides $|-(b-a)| = |b-a|$ as well.

Hence, (b, a) ∈ S, which follows that S is symmetric.

Similarly, if (a, b) ∈ S and (b, c) ∈ S, then $|a-b|$ and $|b-c|$ are both divisible by 3.

That is, $|a-b| = 3m$ i.e., $a - b = \pm 3m$ and $|b-c| = 3n$ i.e., $b - c = \pm 3n$

So, $(a-b)+(b-c)=a-c=\pm 3(m+n)$

$\Rightarrow |a-c|=3(m+n)$

Therefore, $|a-c|$ is divisible by 3.

This shows that S is transitive as (a, c) ∈ S.
Thus, S is an equivalence relation in the set A.
For the equivalence class [4], let $(4, x)\in S$ for all $x \in A$.

That is, $|4-x|$ is divisible by 3. So, clearly $x=1, 4, 7, 10$.

$\therefore [4]=\{1, 4, 7, 10\}$.

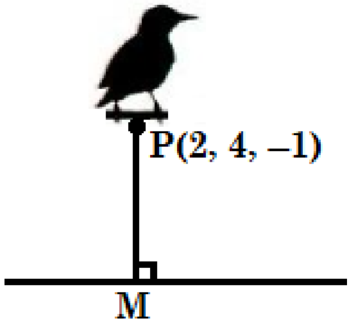
34. Let P(2, 4, –1).

Let $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda \ldots$(i)

The coordinates of any random point on the line (i) is $M(\lambda-5, 4\lambda-3, 6-9\lambda)$.

The d.r's of PM is $\lambda-7, 4\lambda-7, 7-9\lambda$.

If bird wants to reach at point M from its position, then it should fly along PM such that PM is perpendicular to the line (i).



That implies, PM ⊥ line (i) so,

$1(\lambda-7)+4(4\lambda-7)-9(7-9\lambda)=0$

$\Rightarrow 98\lambda=98 \quad \therefore \lambda=1$

$\therefore$ Foot of $\perp^{er}$ drawn from the point P is, M(–4, 1, –3).

$\therefore$ Equation of PM : $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$.

OR

As the d.r.'s of parallel lines are proportional so, the equation of line passing through (2, 3, 2) and parallel to $\vec{r}=(-2\hat{i}+3\hat{j})+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$ is : $\vec{r}=(2\hat{i}+3\hat{j}+2\hat{k})+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$.

Now $\vec{a}_1=-2\hat{i}+3\hat{j},\ \vec{a}_2=2\hat{i}+3\hat{j}+2\hat{k},\ \vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$

$\Rightarrow \vec{a}_2-\vec{a}_1=(2\hat{i}+3\hat{j}+2\hat{k})-(-2\hat{i}+3\hat{j})=4\hat{i}+2\hat{k}$ and,

$$(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 0 & 2\\ 2 & -3 & 6\end{vmatrix}=6\hat{i}-20\hat{j}-12\hat{k}$$

$$\therefore \text{S.D.}=\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$$

$$\Rightarrow \quad =\frac{|6\hat{i}-20\hat{j}-12\hat{k}|}{|2\hat{i}-3\hat{j}+6\hat{k}|}=\frac{\sqrt{36+400+144}}{\sqrt{4+9+36}}=\frac{\sqrt{580}}{7}\text{ units}.$$

35. For the equations $x-y=3,\ 2x+3y+4z=17$ and $y+2z=7$, we may write $AX=B$; where

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\ B=\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}.$$

Now $|A| = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 6$ and $adj.A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$.

Since $AX = B$ $\Rightarrow A^{-1}AX = A^{-1}B$ (Pre-multiplying both sides by A^{-1}

$\Rightarrow IX = A^{-1}B$

Therefore, $X = A^{-1}B$

Now $X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$\therefore$ By equality of matrices, we get : $x = 2,\ y = -1,\ z = 4$.

SECTION E

36. **(i)** Fuel cost $= k(\text{speed})^2$

$\Rightarrow 48 = k \times (16)^2$

$\Rightarrow k = \frac{3}{16}$.

(ii) Total cost of running the train (let C) $= \frac{3}{16}v^2 t + 1200t$

As the distance covered by train is 500 km so, $t = \frac{500}{v}$

$\therefore C = \frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$

$\Rightarrow C = \frac{375}{4}v + \frac{600000}{v}$.

(iii) As $C = \frac{375}{4}v + \frac{600000}{v}$

So, $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ and $\frac{d^2C}{dv^2} = \frac{1200000}{v^3}$

For $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2} = 0$, $v^2 = 6400$ i.e., $v = 80$ km/h

$\because \left(\frac{d^2C}{dv^2}\right)_{\text{at } v=80} = \frac{1200000}{(80)^3} > 0$ so, C will be minimum when $v = 80$ km/h .

OR

(iii) As $C = \frac{375}{4}v + \frac{600000}{v}$

So, $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ and $\frac{d^2C}{dv^2} = \frac{1200000}{v^3}$

For $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2} = 0$, $v^2 = 6400$ i.e., $v = 80$ km/h

$\because \left(\dfrac{d^2C}{dv^2}\right)_{\text{at } v=80} = \dfrac{1200000}{(80)^3} > 0$ so, C will be minimum when $v = 80$ km/h .

Fuel cost for running 500 km, $\dfrac{375}{4}v = \dfrac{375}{4}\times 80 = 7500$ (in ₹).

Also, the total cost for running 500 km, $\dfrac{375}{4}v + \dfrac{600000}{v}$

$$= \frac{375}{4}\times 80 + \frac{600000}{80} = 15000 \text{ (in ₹)}.$$

37. **(i)** $f(x) = mx^2 + 1.2x + 98.6$ being a polynomial function, is differentiable everywhere, hence, the function $f(x)$ is differentiable in $(0, 12)$.

(ii) $f'(x) = 2mx + 1.2$

Since, $x = 6$ is the critical point, $f'(6) = 0$

$\Rightarrow 2m\times 6 + 1.2 = 0$

$\Rightarrow m = -0.1$.

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$

$\Rightarrow f'(x) = -0.2x + 1.2 = -0.2(x-6)$

For $f'(x) = 0$, $-0.2(x-6) = 0$

$\Rightarrow x = 6$.

In the interval	$f'(x)$ is	Conclusion
$(0, 6)$	Positive	f is strictly increasing in (0, 6)
$(6, 12)$	Negative	f is strictly decreasing in (6, 12)

OR

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$

$\Rightarrow f'(x) = -0.2x + 1.2$

For $f'(x) = 0$, $-0.2x + 1.2 = 0 \quad \therefore x = 6$

We have $f(0) = 98.6$, $f(6) = 102.2$, $f(12) = 98.6$.

Clearly, $x = 6$ is the point of absolute maximum and the absolute maximum value of the function $= 102.2$.

Also $x = 0, 12$ both are the points of absolute minimum and the absolute minimum value of the function $= 98.6$.

38. Note that, $P(E_1) = \dfrac{3}{5}$, $P(E_2) = \dfrac{2}{5}$, $P(E|E_1) = 1$, $P(E|E_2) = \dfrac{1}{3}$.

(i) $\sum_{k=1}^{k=2}\left[P(E|E_k)\ P(E_k)\right] = P(E|E_1)\ P(E_1) + P(E|E_2)\ P(E_2)$

$$\Rightarrow \qquad = 1\times\frac{3}{5} + \frac{1}{3}\times\frac{2}{5} = \frac{11}{15}.$$

(ii) By Bayes' theorem, $P(E_1|E) = \dfrac{P(E|E_1)\ P(E_1)}{P(E|E_1)\ P(E_1) + P(E|E_2)\ P(E_2)}$

$$\therefore P(E_1|E) = \frac{1\times\frac{3}{5}}{1\times\frac{3}{5} + \frac{1}{3}\times\frac{2}{5}} = \frac{9}{11}.$$

ABOUT THE AUTHOR

O.P. GUPTA having taught math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn math without being afraid. Undoubtedly his mathematics books are best sellers on Amazon and Flipkart. His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

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